

Problem Set 4

Due 6pm Friday October 11

0. **Reading:** Sections 17.1, 17.2, 17.5, 17.6 of Thorne and Blandford

1. Space Shuttle Re-Entry

(a) Assume the atmosphere is approximately an adiabatic ideal gas. For a highly supersonic spacecraft moving at speed V , show that the post-shock temperature is

$$T_2 \approx \frac{2(\gamma - 1)\mu m_p V^2}{(\gamma + 1)^2 k_B}. \quad (1)$$

(b) When the space shuttle re-entered the atmosphere, it reached a speed of ~ 9 km/s at 70 km altitude. At this altitude, the sound speed of the atmosphere is ~ 280 m/s, the atmospheric pressure is about 0.4 atm, and the mean molecular weight is $\mu \sim 10$ (appropriate to dissociated air). Calculate (i) the pre-shock Mach number; (ii) the post-shock Mach number; does it depend on any pre-shock properties? (iii) the post-shock pressure (in atm); (iv) the post-shock temperature in Kelvin. (This temperature is high enough to ionize the air partially, leading to radio communication blackout for about 12 minutes.)

2. The de Laval Nozzle

This problem is relevant for a jet engine (and later on, astrophysical jets), in which hot and pressurized gas flows through a horizontal tube of varying cross-sectional areas designed to accelerate the gas to supersonic speed. Assume the gas follows steady, adiabatic and irrotational flows. Ignore gravity.

(a) From the Bernoulli's theorem, show the gas density ρ and flow speed v obey

$$\frac{d\rho}{\rho} = -M^2 \frac{dv}{v}, \quad (2)$$

where M is the Mach number.

(b) From mass conservation for a steady flow, write down an equation relating ρ , v , and the cross-sectional area A . Use this relation and part (a) to show

$$\left(M^2 - 1\right) \frac{dv}{v} = \frac{dA}{A}, \quad \left(M^{-2} - 1\right) \frac{d\rho}{\rho} = \frac{dA}{A}. \quad (3)$$

(c) Describe *qualitatively* how you would vary the tube's cross section in order to change the flow's speed *smoothly* from subsonic to supersonic. Show that the sonic transition occurs at the "throat" of the nozzle at which A is a minimum.