Astro/Phys 202: Astrophysical Fluid Dynamics October 3 2025

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Problem Set 4

Due 6pm Friday October 10

0. **Reading**: Sections 17.1, 17.2, 17.5, 17.6 of Thorne and Blandford

1. **Space Shuttle Re-Entry**

(a) Assume the atmosphere is approximately an adiabatic ideal gas. For a highly supersonic spacecraft moving at speed V, show that the post-shock temperature is

$$T_2 \approx \frac{2(\gamma-1)\mu m_p V^2}{(\gamma+1)^2 k_B} \,. \tag{1}$$

(b) When the space shuttle re-entered the atmosphere, it reached a speed of $\sim$ 8 km/s at 80 km altitude. At this altitude, the sound speed of the atmosphere is $\sim$ 280 m/s, the atmospheric pressure is about 0.4 atm, and the mean molecular weight is $\mu \sim 10$ (appropriate to dissociated air). Calculate (i) the pre-shock Mach number; (ii) the post-shock Mach number; does it depend on any pre-shock properties? (iii) the post-shock pressure (in atm); (iv) the post-shock temperature in Kelvin. (This temperature is high enough to ionize the air partially, leading to radio communication blackout for about 12 minutes.)

2. **Jets**

This problem is relevant for a jet engine (and later on, astrophysical jets), in which hot and pressurized gas flows through a horizontal tube of varying cross-sectional areas designed to accelerate the gas to supersonic speed. Assume the gas follows steady, adiabatic and irrotational flows. Ignore gravity.

(a) From the Bernoulli's theorem, show the gas density ρ and flow speed v obey

$$\frac{d\rho}{\rho} = -M^2 \frac{dv}{v} \,, \tag{2}$$

where M is the Mach number.

(b) From mass conservation for a steady flow, write down an equation relating ρ, v, and the cross-sectional area A. Use this relation and part (a) to show

$$\left(M^2 - 1\right) \frac{dv}{v} = \frac{dA}{A} \,, \quad \left(M^{-2} - 1\right) \frac{d\rho}{\rho} = \frac{dA}{A} \,. \tag{3}$$

(c) Describe *qualitatively* how you would vary the tube's cross section in order to change the flow's speed *smoothly* from subsonic to supersonic. Show that the sonic transition occurs at the "throat" of the nozzle at which A is a minimum.