Astro/Phys 202: Astrophysical Fluid Dynamics October 18 2024

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Problem Set 6

Due 6pm Friday October 25

1. **Estimate Jeans Length**

At typical sea-level conditions, the density of air is 1.2×10^{-3} g cm^{-3} and the speed of sound is 340 m sec^{-1}. Estimate the Jeans length in km. Comment on how it compares with the thickness of the atmosphere and if you expect Jeans instabilities to occur.

2. **Is the Milky Way Disk Stable?**

The Milky Way's disk is thin and mostly made of stars. Near the solar neighborhood, the local surface density of the stars is about $74 M_\odot$ pc^{-2}, the radial velocity dispersion is about 45 km s^{-1}, and the epicycle frequency is 36 km s^{-1} kpc^{-1}. Do you expect the stellar disk in the solar neighborhood to be stable or unstable against gravitational collapse?

3. **No More Jeans Swindle: a Rotating System**

The Jeans instability can be analyzed exactly, without invoking the Jeans swindle, in certain cylindrical rotating systems. Consider a homogeneous, self-gravitating fluid of density ρ_0, contained in an infinite cylinder of radius R_0. The cylinder walls and the fluid rotate at uniform angular speed $\vec{\Omega} = \Omega \hat{z}$, where $\hat{z}$ lies along the axis of the cylinder. The Euler equation for this rotating system is

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla} P - \vec{\nabla}\phi - 2\vec{\Omega} \times \vec{v} + \Omega^2 (x\hat{x} + y\hat{y}) \,, \tag{1}$$

where the additional terms are the Coriolis and centrifugal forces.

(a) From the Poisson equation, show that the gravitational force per unit mass inside the cylinder is

$$-\vec{\nabla}\phi_0 = -2\pi G \rho_0 (x\hat{x} + y\hat{y}) \,. \tag{2}$$

(b) From the Euler equation above, write down an expression for the same quantity, $-\vec{\nabla}\phi_0$, assuming the fluid is in equilibrium with zero velocity and no pressure gradients. What condition must Ω obey to eliminate the Jeans swindle?

(c) Let $R_0 \to \infty$ so that the boundary condition due to the wall can be neglected. From the perturbed fluid equations, find the dispersion relation for waves propagating **parallel** to the rotation axis $\hat{z}$. Discuss if these waves are stable.

(d) Find the dispersion relation for waves propagating **perpendicular** (you may pick $\hat{x}$ without loss of generality) to the rotation axis $\hat{z}$. Discuss if these waves are stable.