

Problem Set 6

Due 6pm Friday October 24

1. Is the Milky Way Disk Stable?

The Milky Way's disk is thin and mostly made of stars. Near the solar neighborhood, the local surface density of the stars is about $72M_{\odot} \text{ pc}^{-2}$, the radial velocity dispersion is about 43 km s^{-1} , and the epicycle frequency is $36 \text{ km s}^{-1} \text{ kpc}^{-1}$. Do you expect the stellar disk in the solar neighborhood to be stable or unstable against gravitational collapse?

2. Estimate Jeans Length

At typical sea-level conditions, the density of air is $1.1 \times 10^{-3} \text{ g cm}^{-3}$ and the speed of sound is 340 m sec^{-1} . Estimate the Jeans length in km. Comment on how it compares with the thickness of the atmosphere and if you expect Jeans instabilities to occur.

3. No More Jeans Swindle: a Rotating System

The Jeans instability can be analyzed exactly, without invoking the Jeans swindle, in certain cylindrical rotating systems. Consider a homogeneous, self-gravitating fluid of density ρ_0 , contained in an infinite cylinder of radius R_0 . The cylinder walls and the fluid rotate at uniform angular speed $\vec{\Omega} = \Omega \hat{z}$, where $\hat{z}$ lies along the axis of the cylinder. We analyzed the fluid equations for this rotating system in class.

(a) From the Poisson equation, show that the gravitational force per unit mass inside the cylinder is

$$-\vec{\nabla}\phi_0 = -2\pi G\rho_0(x\hat{x} + y\hat{y}). \quad (1)$$

(b) From the Euler equation, write down an expression for the same quantity, $-\vec{\nabla}\phi_0$, assuming the fluid is in equilibrium with zero velocity and no pressure gradients. What condition must Ω obey to eliminate the Jeans swindle?

(c) Let $R_0 \rightarrow \infty$ so that the boundary condition due to the wall can be neglected. From the perturbed fluid equations, find the dispersion relation for waves propagating **parallel** to the rotation axis $\hat{z}$. Discuss if these waves are stable.

(d) Find the dispersion relation for waves propagating **perpendicular** (you may pick $\hat{x}$ without loss of generality) to the rotation axis $\hat{z}$. Discuss if these waves are stable.