Astro/Phys 202: Astrophysical Fluid Dynamics October 25 2025

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Problem Set 7

Due 6pm Friday November 7 2025

0. **Reading**: Sec. 13.7 of Thorne & Blandford.

1. **Viscosity and Reynolds Number**

(a) A typical artery has a radius of ~ 1 mm and length of ~ 1 m. You often hear that the desired level of blood pressure is ~ 110 mm (of mercury) for the systolic pressure and 70 mm for the diastolic pressure. The difference of ~ 40 mm of mercury, or $\sim 5\times10^3$ N/m^2, is a good estimate for the pressure difference in an artery. Estimate the mass and volume flow rates of blood in an artery in kg/s and in m^3/s, respectively. Compare your answer with the mean pump rate of ~ 5 liters per minute in a healthy adult. Is it similar or very different? Note that we have assumed the walls of an artery to be rigid, which fortunately is quite elastic.

(b) Estimate the Reynolds number for the following flows. An order of magnitude estimate is sufficient, but justify or give references to the numbers you use. (i) Flow past the wing of a 777 jet at 0.8 Mach; (ii) A blue whale in the ocean; (iii) Plankton in the ocean; (iv) A thick layer of maple syrup draining off a spoon.

2. **Viscous Flow between Two Plates**

A viscous fluid flows steadily (no time dependence) in the z-direction, with the flow confined between two plates that are parallel to the $x-z$ plane and are separated by a distance 2a. Show that the flow's velocity field is

$$v_z = -\frac{dP}{dz}\frac{a^2}{2\eta}\left[1-\left(\frac{y}{a}\right)^2\right], \tag{1}$$

and the mass flow rate per unit width of the plates is

$$\frac{dM}{dtdx} = -\frac{dP}{dz}\frac{2a^3}{3\nu}. \tag{2}$$

Here dP/dz (which is negative) is the pressure gradient along the direction of flow.

3. **Keplerian Accretion Disk**

For an initial surface mass distribution of the form

$$\Sigma(R,0) = \frac{m}{2\pi R_0}\delta(R-R_0), \tag{3}$$

one can show that the solution to the evolution equation for a Keplerian disk is

$$\Sigma(x,\tau) = \frac{m}{\pi R_0^2}\frac{1}{\tau x^{1/4}}\, e^{-\frac{1+x^2}{\tau}}\, I_{1/4}(2x/\tau), \tag{4}$$

where I_p is the modified Bessel function of order p, and the dimensionless variables are $x \equiv R/R_0$ and $\tau = (12\nu/R_0^2)t$.

(a) Show that the corresponding solution for the radial velocity is given by

$$v_R(x,\tau) = -\frac{3\nu}{R_0}\frac{\partial}{\partial x}\left[\frac{1}{4}\ln x - \frac{1+x^2}{\tau} + \ln I_{1/4}(2x/\tau)\right] . \tag{5}$$

(b) Show that for $2x \gg \tau$, one has

$$v_R \approx \frac{3\nu}{R_0}\left[\frac{1}{4x} + \frac{2x}{\tau} - \frac{2}{\tau}\right] . \tag{6}$$

Is the material in the disk outside the initial radius R_0 flowing inward or outward?

(c) Show that for $2x \ll \tau$, one has

$$v_R \approx -\frac{3\nu}{R_0}\left[\frac{1}{2x} - \frac{2x}{\tau}\right] . \tag{7}$$

Within which region of the disk is the material flowing inward?

(d) On a single figure, plot $\Sigma(x,\tau)$ (in units of $m/(\pi R_0^2)$) vs. x at four times (each quadruping the previous time): $\tau = 0.004$, 0.016, 0.064, and 0.256. Comment (qualitatively) on how your plot agrees or disagrees with what you found in parts (b) and (c).