C228: Extragalactic Astronomy and Cosmology January 29 2025

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Problem Set 1

Due: 6pm Friday February 7 2025

Homework Policy: **No late homework**. You may discuss the problem set with your classmates but you must solve and write up the solutions independently and make the plots yourself. All plots should be done on a computer.

0. **Reading**: Sections 1.1-1.3 of Weinberg "The Expansion of the Universe"

1. **When did the Universe Start to Accelerate?**

The expansion of the Universe is currently accelerating, but it was not always so.

(a) Derive an expression for the redshift at which the deceleration turned into acceleration. Your expression should contain only fundamental cosmological parameters at the present day such as $\Omega_{0,m}$, $\Omega_{0,\Lambda}$, H_0, k etc.

(b) Compute this redshift for the currently favored cosmology: a flat Universe with $\Omega_{0,m} = 0.31$, $\Omega_{0,\Lambda} = 0.69$, and $H_0 = 70$ km s^{-1} Mpc^{-1}.

(c) Compute the redshift for the matter-Λ equality time, which is defined to be the epoch at which the energy density in matter is equal to that in the cosmological constant. Comment on how this redshift is similar to or different from what you found in part (b).

2. **The Hubble Parameter**

The Hubble "constant" in general is not a constant in time. You will derive its time evolution here.

(a) Show that the curvature parameter k in the Friedmann equation can be written in terms of the present-day H_0 and Ω_0 as

$$k = H_0^2(\Omega_0 - 1)\,. \tag{1}$$

(b) Using (a) and the Friedmann equation, show that the Hubble parameter is a function of the scale factor a (which in turn depends on time) of the form

$$H^2(a) = H_0^2\left(\frac{\Omega_0}{a^{3+3w}} + \frac{1-\Omega_0}{a^2}\right)\,. \tag{2}$$

3. **The Density Parameter**

We derived in class that the density parameter Ω depends on the scale factor a as

$$1 - \Omega(a) = \frac{1 - \Omega_0}{1 - \Omega_0 + \Omega_{0,\Lambda} a^2 + \Omega_{0,m} a^{-1} + \Omega_{0,r} a^{-2}}\,, \tag{3}$$

where $\Omega_0 = \Omega(a = 1) = \Sigma_i \Omega_{0,i}$ is the present-day value of the total density parameter $\Omega(a)$.

(a) Compute $\Omega(a)$ at $a = 10^{-3}$ for four universes with different compositions: $(\Omega_{0,m}, \Omega_{0,\Lambda}) = (0.31, 0.0)$, $(0.31, 0.69)$, $(1.0, 0.0)$, and $(5.0, 0.0)$. (The universe at $a = 10^{-3}$ is matter-dominated to a good approximation, so you can ignore radiation.) On a single figure, plot $\Omega(a)$ for the four cases. (Use logarithmic scales for both axes and the range 10^{-3} to 1 for a.) What trend do you see?

(b) From the parametric solutions to the Friedmann equation, show that $a(t) \propto t^{\beta}$ at *small* t for both open and close models. (Again, consider only matter-dominated era.) Find the value of β and compare it to the value for the flat model. Is your answer consistent with (a)?