C228: Extragalactic Astronomy and Cosmology February 15 2025

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Problem Set 3

Due 6pm Friday February 21

1. **Hubble Diagram I: Distance vs. Redshift**

(a) Show that for arbitrary $\Omega_{0,m}$ and $\Omega_{0,\Lambda}$ (ignoring radiation), the comoving distance r and redshift z are related by

$$r = |k|^{-1/2}\, sinn \left\{ \frac{c|k|^{1/2}}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_{0,m}(1+z')^3 + \Omega_{0,\Lambda} + (1-\Omega_{0,m}-\Omega_{0,\Lambda})(1+z')^2}} \right\}, \quad (1)$$

where $sinn = \sin$ for $k > 0$, $sinn = \sinh$ for $k < 0$, and $sinn = 1$ (i.e., absent) for $k = 0$.

(b) For small redshift, expand the expression above and show that for all three cases of k,

$$r = \frac{c}{H_0}\left[z - \frac{1}{2}(1+q_0)\, z^2 \right] + O(z^3)\,, \quad (2)$$

where q_0 is the deceleration parameter. As the next problem will show, this is the key equation used in the measurements of q_0.

2. **Hubble Diagram II: Supernova Data**

You derived the relation between the comoving distance and redshift above. In practice, astronomers often use a logarithmic scale, the *distance modulus* μ, to measure distances. The distance modulus is related to the luminosity distance, d_L, by

$$\mu \equiv 5\, \log_{10} \frac{d_L}{10\,\mathrm{parsec}}\,. \quad (3)$$

(a) Make a plot of μ vs. z for three cosmological models: $(\Omega_{0,m}, \Omega_{0,\Lambda}) = (0.27, 0.73)$, (0.27, 0.0), and (1.0, 0.0). Assume $h = 0.70$ for the Hubble parameter.

(b) The course website (click at “Data” after “Problem Set 3”) lists μ and z for 41 supernovae from Table 4 of Riess et al (2007). Add these data points to your plot. Compare your plot to their Figure 6 and comment (e.g. are they similar? If not, why not?).