

Problem Set 4

Due 6pm Friday February 28

1. The Most Distant Galaxies

The highest redshift galaxy observed by JWST is claimed to be at $z \approx 14$. The most distant photons we have detected are from the cosmic microwave background at $z \approx 1100$. Compute the ages of the universe when the light that we receive today was emitted from these two sources for three cosmological models: $(\Omega_{0,m}, \Omega_{0,\Lambda}) = (0.3, 0.7)$, $(0.3, 0.0)$, and $(1.0, 0.0)$. Assume $h = 0.7$ for the Hubble parameter.

2. The Curious Behavior of the Angular Diameter

(a) For an object of physical size L at redshift z , write down a general expression for its angular diameter as a function of L , z , $\Omega_{0,m}$, and H_0 (assume $\Omega_\Lambda = 0$ for simplicity). Rewrite the formula so that the angular diameter is in units of h arcsec (where $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$) and L in units of kpc.

(b) The physical size of the luminous part of a Milky-Way-like galaxy is about 15 kpc. On log-log scales, plot the angular diameter of such a galaxy versus redshift ($0.01 \leq z \leq 15$) for three values of $\Omega_{0,m}$: 0.3, 1.0, and 3.0. **Make sure you have three curves on a single figure and not three separate figures.** Comment on any interesting features in your curves. (Again assume $\Omega_\Lambda = 0$.)

3. Age of the Universe

The age of a universe, t_0 , depends on cosmological parameters.

(a) For $\Omega_\Lambda = 0$ models, plot t_0 in units of h^{-1} Gyr versus the matter density parameter $\Omega_{0,m}$, for $0 \leq \Omega_{0,m} \leq 3$. What is the general trend of t_0 as the matter density increases?

(b) On the same plot, add a curve for t_0 vs. $\Omega_{0,m}$ for flat models (i.e. $\Omega_{0,m} + \Omega_{0,\Lambda} = 1$). For a given $\Omega_{0,m}$ (where $\Omega_{0,m} < 1$), is the age for a flat universe with non-zero $\Omega_{0,\Lambda}$ smaller or larger than an open universe without a cosmological constant?

(c) Consider the flat models in (b). Plot H_0 , in units of $\text{km s}^{-1} \text{ Mpc}^{-1}$, versus $\Omega_{0,m}$, for 3 values of t_0 : 11, 13.8, 19 Gyr. Current observations find $H_0 \approx 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and the oldest objects in the universe is *at least* 11 Gyr old. Given these, what is the constraint on $\Omega_{0,m}$ implied by your curves? Many theorists favored $\Omega_{0,m} = 1$ before the discovery of an accelerating universe. What would be the constraint on H_0 implied by $t_0 > 11.5$ Gyr if $\Omega_{0,m} = 1$? How is it compared with the observed H_0 ?