C228: Extragalactic Astronomy and Cosmology — February 28 2025

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Problem Set 5

Due 6pm Friday March 7

0. **Reading:** Big Bang Nucleosynthesis from Review of Particle Physics

1. Ω in Photon Background Radiation

The present temperature of the cosmic background photons is known to exquisite precision: $T_0 = 2.725 \pm 0.001$ Kelvin (Mather et al 1999).

(a) Calculate the present number density of photons (in cm^{-3}) in this radiation. (Don't worry about the error bars.)

(b) Calculate $\Omega_{0,\gamma}$, the present ratio of the energy density in photons to the critical density. (Express your answer in terms of the Hubble parameter h.) Is the universe today radiation or matter dominated?

2. Ω in Neutrino Background Radiation

Let's repeat Problem 1 for the cosmic neutrinos.

(a) What is the present-day temperature of the neutrino background?

(b) Calculate the present-day number density of the electron neutrinos ν_e and their anti-particles $\bar{\nu}_e$ in cm^{-3}.

(c) If the masses of all three species of neutrinos (i.e. ν_e, ν_μ, ν_τ) are much less than 1 MeV, explain how the number densities of ν_μ and ν_τ compare to your answer for ν_e in part (b).

(d) For what (approximate) range of mass (in eV) would neutrinos behave like relativistic particles today? Assume neutrinos are relativistic today. The total background radiation then consists of photons, 3 species of left-handed neutrinos, and their corresponding anti-neutrinos. Calculate $\Omega_{0,r}$, the present-day density parameter for this radiation.

3. Ω in Baryons

Let's look at baryons now. Show that $\Omega_{0,B}$, the contribution of baryons to the present-day density parameter, is related to the baryon-to-photon ratio, $\eta \equiv n_B/n_\gamma$, by

$$\Omega_{0,B}\, h^2 = \beta\, \eta\,, \qquad (1)$$

and calculate β using your result in Problem 1 above. Current measurements of light elemental abundances indicate $\eta = (6.04 \pm 0.118) \times 10^{-10}$ (95% CL). What is the implied $\Omega_{0,B}$?

4. Equality Time

(a) **Radiation-Matter Equality**:
From your answer to Problem 2(d) above, calculate the equality redshift, z_{eq}, at which the energy density in matter equals that in the radiation. (Leave $\Omega_{0,m}$ and h as free variables in your answer.)

For the currently favored model of $\Omega_{0,m} = 0.31$, $\Omega_{0,\Lambda} = 0.69$, and $h = 0.7$, what is z_{eq}? Approximately how old was the universe at this redshift?

(b) **Matter-Λ Equality**:
In problem 1(c) of Problem Set 1, you calculated the matter-Λ equality redshift, the redshift at which the energy density in matter equals that in the cosmological constant. Using this information and part (a) above, make a simple sketch showing how the energy density in radiation, matter, and Λ evolves with the expansion factor a (use log-log scale), and indicate the crossing times.