C228: Extragalactic Astronomy and Cosmology March 19 2025

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Problem Set 7

Due 6pm Friday April 4

0. **Reading**: Dark Matter from Review of Particle Physics

1. **Mass versus Light**

A primary goal of many galaxy surveys is to measure the distribution of the luminosities of galaxies, which has been found to be well parameterized by a luminosity function of the form

$$\phi(L)dL = \phi_* \left(\frac{L}{L_*}\right)^\alpha e^{-L/L_*} d\left(\frac{L}{L_*}\right) , \tag{1}$$

where $\phi(L)dL$ is the number density of galaxies with luminosity between L and $L + dL$, and ϕ_*, L_*, and α are parameters to be determined from galaxy surveys.

(a) Show that the luminosity density of galaxies, j, is related to ϕ_*, L_*, and α by a simple algebraic expression. Derive this expression. (Hint: If your answer contains an integral, you haven't tried hard enough.)

(b) A recent galaxy survey measures $\phi_* = 0.0165\, h^3$ Mpc^{-3}, $\alpha = -1.13$, and $L_* = 1.26 \times 10^{10}\, h^{-2} L_\odot$. Calculate j. What fraction of this luminosity density is contributed by galaxies fainter than L_*? (Leave h as a free parameter.)

(c) A classical technique for quantifying the amount of dark vs. luminous matter is to determine the mass-to-light ratio, M/L (in units of solar $M_\odot/L_\odot$). Assuming the luminosity density from part (b) is representative for the universe, calculate the value of M/L (in units of $M_\odot/L_\odot$) required to yield $\Omega_m = 0.3$. (The typical M/L value for individual galaxies is in the range of a few to ~ 10.)

2. **No More Jeans Swindle: an Expanding Medium**

For an expanding non-relativistic fluid, show that one no longer has to invoke Jeans swindle, and instead, you obtain

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3} G\rho \,, \tag{2}$$

where a is the scale factor of an isotropic and homogeneous expanding universe and ρ is the average (i.e. unperturbed) density of the universe. Compare this equation with the equation for $\ddot{a}$ that we discussed near the beginning of the semester. If there is any difference, comment on the origin of the difference.

3. **Estimate Jeans Length and Mass**

At about 400,000 years after the big bang, the universe became cold enough for the baryons (i.e. ionized hydrogen, helium) to form neutral atoms. This time is called "decoupling" since the photons traveled (mostly) freely afterwards. Decoupling occurred at a temperature of about 3000 Kelvin, and a scale factor of $a \approx 10^{-3}$ (where $a = 1$ today).

(a) After decoupling, baryons in the universe can be modeled as an adiabatic ideal gas with $\gamma \approx 5/3$. Estimate (i) the sound speed of baryons (in km/s), (ii) Jeans length (in physical kpc), and (iii) Jeans mass (in solar masses) *right after* decoupling has occurred. The mean density of baryons (in comoving coordinates) is about 4×10^{-28} kg/m^3. How does your Jeans mass compare to the mass of a massive star, a globular cluster, the Milky Way, and a galaxy cluster? Comment on your result.

(b) Before decoupling, the baryons were tightly coupled to the photons and behaved as a single fluid. During this time, the sound speed in the baryons was highly relativistic, $v_s \approx c/\sqrt{3}$, where c is the speed of light. Use the ratio of this sound speed to the one you found in (b), and scale the Jeans length and Jeans mass in (b) to find the Jeans length and Jeans mass in baryons *right before* decoupling. How does your Jeans mass compare to the mass of a massive star, a globular cluster, the Milky Way, and a galaxy cluster? Comment on your result.