C228: Extragalactic Astronomy and Cosmology April 14 2025

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Problem Set 8

Due 6pm Friday April 25

1. **Structure Formation I: Radiation Era**

In class we derived the time evolution of density perturbations in non-relativistic matter δ_m (above the Jeans length) in the matter-dominated era. Here we will find the behavior of δ_m in the radiation-dominated era. Assume a flat, $\Omega_\Lambda = 0$ universe for simplicity.

(a) Assume that the *perturbation* in radiation, δ_r, is negligible. Show that the *linearized* evolution equation for δ_m can be written as

$$\frac{d^2\delta_m}{dy^2} + \frac{2+3y}{2y(1+y)}\frac{d\delta_m}{dy} - \frac{3}{2y(1+y)}\delta_m = 0\,, \tag{1}$$

where $y \equiv \rho_m/\rho_r = a/a_{eq}$ and a_{eq} is the equality time.

(b) At late time ($y \gg 1$), compare your solutions for δ_m with those derived in class.

(c) Verify that $\delta_m \propto y + 2/3$ is a solution to eq. (1) in general. Can δ_m grow much in the radiation-dominated era?

2. **Structure Formation II: $\Omega_m \neq 1$ Models**

(a) **Open Models**: In class we derived the time evolution of the matter density perturbation for an open model with $\Omega_\Lambda = 0$. Plot the growing solution $\delta_m(a)$ on a log-log scale for $a = 0.001$ to 1.0 for four models with $\Omega_{0,m} = 0.01, 0.1, 0.3$ and 1.0 (assuming $\Omega_\Lambda = 0$). Normalize all your curves to $\delta_m(a = 0.001) = 0.001$. **Make sure to plot all curves on a single figure so you can compare them.**

Briefly comment on how the growth of perturbations depends on $\Omega_{0,m}$. If the four models you plotted all produce the same level of perturbations today (that matches the observed number density of galaxies, for example), how do you expect (qualitatively) the amount of structures to differ in the four models at redshift 5?

(b) **Lambda Models**: The general solution for the growing mode of δ_m can be shown to be

$$\delta_m(a) \propto \frac{\dot{a}}{a}\int_0^a \frac{da'}{\dot{a}'^3}\,. \tag{2}$$

(See p. 468 of Peacock for a derivation.) Add $\delta_m(a)$ to your earlier plot for two flat models: $(\Omega_{0,m}, \Omega_{0,\Lambda}) = (0.3, 0.7)$ and $(0.1, 0.9)$. Comment on how the growth of δ_m changes with $\Omega_{0,m}$ and $\Omega_{0,\Lambda}$. **Make sure to plot all curves on the same figure so you can compare them.**

If the three models $(\Omega_{0,m}, \Omega_{0,\Lambda}) = (1, 0), (0.3, 0.7)$ and $(0.3, 0.0)$ all produce the same level of fluctuations today (that matches the observed number density of collapsed objects, for example), how do you expect (qualitatively) the amount of structures to differ in the three models at redshift 7?