Astro/Phys 228: Extragalactic Astronomy and Cosmology April 25 2025

C.–P. Ma

Problem Set 9

Due 6pm Tuesday May 6

0. **Reading**: Inflation from Review of Particle Physics.

1. **Course Evaluation**: Please fill in the online course evaluation.

2. **The Flatness Problem**

In Problem Set 1, we computed the density parameter at recombination time ($a = 0.001$) for four models: $(\Omega_{0,m}, \Omega_{0,\Lambda}) = (0.31, 0.0)$, $(0.31, 0.69)$, $(1.0, 0.0)$, and $(5.0, 0.0)$. Repeat this calculation for the four models at the Planck time (when $T \sim 10^{19}$ GeV). Briefly explain why this is a problem.

3. **The Horizon Problem**

The particle horizon at time t, $d_H(t)$, is the *physical* distance that light has traveled since $t = 0$; it defines the maximum distance for causal communication. The expression for d_H can be solved exactly in the matter-dominated era for $\Omega_\Lambda = 0$:

$$d_H(z) = \begin{cases} \frac{c}{H_0\sqrt{1-\Omega_{0,m}}}(1+z)^{-1} \cosh^{-1}\left[1 + \frac{2(1-\Omega_{0,m})}{(1+z)\Omega_{0,m}}\right], & \Omega_{0,m} < 1 \\ \frac{2c}{H_0}(1+z)^{-3/2}, & \Omega_{0,m} = 1 \\ \frac{c}{H_0\sqrt{\Omega_{0,m}-1}}(1+z)^{-1} \cos^{-1}\left[1 - \frac{2(\Omega_{0,m}-1)}{(1+z)\Omega_{0,m}}\right], & \Omega_{0,m} > 1 \end{cases}$$

(a) Derive each of these three expressions from the definition of the particle horizon.

(b) What is the present horizon size (in Mpc) if $\Omega_{0,m} = 0.3$? $\Omega_{0,m} = 1.0$? $\Omega_{0,m} = 3.0$?

(c) Show that at high redshifts $1 + z \gg \Omega_{0,m}^{-1}$, a good approximation for all three cases is given by

$$d_H(z) \approx \frac{2c}{\sqrt{\Omega_{0,m}}H_0}(1+z)^{-3/2}\,. \quad (1)$$

(You may find the identity $\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1})$ useful.)

(d) Show that the *comoving* size of the particle horizon at the time of photon decoupling can be written as $\lambda_{dec} = \beta\,(\Omega_{0,m}\,h^2)^{-1/2}$, and compute the value of β in Mpc.

(e) Estimate the angular size (in degrees) subtended by λ_{dec} on the sky today. This is called the horizon problem. Why is this a *problem*? (The really relevant scale should be the *acoustic* horizon of the baryon-photon fluid rather than λ_{dec} here, but the two length scales are similar.)

4. **Two-Point Correlation Function**

We discussed the two-point correlation function and the power spectrum of the density perturbation field.

(a) Show that $\xi(r)$ is the Fourier transform of the power spectrum $P(k)$.

(b) For a power-law power spectrum $P(k) \propto k^n$, derive the dependence of $\xi(r)$ on r.