Astro 7A – Problem Set 2

1 Martian Speeding Ticket

Adapted from Saul Rappaport's Introductory Astrophysics class.

A monochromatic (single frequency) beam is aimed at Mars from the Earth with a frequency of 1000 MHz. By the time the beam arrives at Mars, it covers the entire Martian surface. Mars reflects the beam back to Earth. The echo, detected at Earth, is observed to have a range of frequencies spanning a bandwidth (maximum frequency - minimum frequency) of 1.6 kHz.

Deduce from this observation the rotation rate of Mars. Mars has an equatorial radius of 3400 km.

2 "Staring at the Sky, Staring at the Sun"

The Sun's apparent visual magnitude is $m_{\mathrm{app},\odot} = -26.83$ and its absolute visual magnitude is $m_{\mathrm{abs},\odot} = 4.83$.

Vega has (is assigned) an apparent visual magnitude of $m_{\mathrm{app,Vega}} = 0$ and an absolute visual magnitude of $m_{\mathrm{abs,Vega}} = 0.58$.

(a) Calculate the *flux* ratio (in the visual passband) between the Sun and Vega ($F_\odot/F_{\mathrm{Vega}}$). Give both a symbolic and numerical answer. Which object is *apparently brighter* (duh)?

(b) Calculate the *luminosity* ratio (in the visual passband) between the Sun and Vega ($L_\odot/L_{\mathrm{Vega}}$) using only the information given above. Give both a symbolic and numerical answer. Which object is *more luminous*?

(c) Given the radius of the Sun ($R_\odot$) and the distance to the Sun (1 AU), what *solid angle* does the Sun subtend? Express in both steradians and square arcseconds.

(d) What is the visual *surface brightness* of the Sun? Express in magnitudes per square arcsecond. Hint: the answer is NOT merely $m_{\mathrm{app},\odot} = -26.83$ divided by your answer in (c).

3 Binning in Frequency vs. Binning in Wavelength

Spectra look different depending on whether they are computed per wavelength or per frequency. The spectrum of a blackbody when binned in frequency space (B_ν) does not equal the spectrum of the same blackbody when binned in wavelength space (B_λ). The often-used (and much abused) Wien displacement law (a.k.a. Wien peak law) for blackbodies actually refers to the peak of B_λ, not of B_ν (a form of physics chauvinism, totally unjustifiable). The frequency where B_ν peaks is not merely the speed of light c divided by the wavelength where B_λ peaks. The entire shapes of B_λ and B_ν differ.

These facts may be a little confusing at first (I remember being confused myself), but they are just simple consequences of how we choose to bin photons when plotting spectra. We can either choose to bin the photons in bins of equal frequency width, or we can choose to bin the photons in bins of equal wavelength width. The spectrum will, in general, look different depending on our choice.

(a) Suppose Astronomer A measures the spectrum of a point source (NOT a blackbody). This Astronomer collects photons from the object in $N = 10$ equally spaced bins in wavelength between $\lambda_{\rm min} = 1000$ and $\lambda_{\rm max} = 10000$ Angstroms. Each bin has a wavelength width of $\Delta\lambda = (\lambda_{\rm max} - \lambda_{\rm min})/N = 900$ Angstroms. So the astronomer collects photons in the first bin having wavelengths between 1000 and 1900 Angstroms; photons in the second bin having wavelengths between 1900 and 2800 Angstroms; and so on.

Photons are collected from the source for a total of $\Delta t = 50$ seconds. The telescope has a collecting area of $\Delta A = 20000$ square cm.

When Astronomer A tallies up the ENERGY contained in each bin, it is discovered that each bin contains the same amount of energy $\Delta E = 90$ erg. From this result, plot the *flux density in wavelength space* F_λ between $\lambda_{\rm min}$ and $\lambda_{\rm max}$. Label your axes. Give your answer in cgs units.

(b) Astronomer B performs almost exactly the same experiment. The only difference is that Astronomer B collects photons in $N = 10$ equally spaced bins in FREQUENCY space, from $\nu_{\rm min} = c/\lambda_{\rm max}$ and $\nu_{\rm max} = c/\lambda_{\rm min}$, where c is the speed of light. The bins for Astronomer B have equal frequency widths $\Delta\nu$.

Given (a), plot the *flux density in frequency space* F_ν between $\nu_{\rm min}$ and $\nu_{\rm max}$. Label your axes. Give your answer in cgs units.

Hint: $\Delta\nu \neq c/\Delta\lambda$. Ask yourself why.

(c) Does F_λ look different from F_ν (remember it is one and the same object, observed under identical conditions. The only difference is how Astronomers A and B divide up the photons when plotting their results). Does F_λ have a peak? If so, specify $\lambda_{\rm peak}$. Does F_ν have a peak? If so, specify $\nu_{\rm peak}$.

(d) How much TOTAL energy is collected by Astronomer A? Astronomer B?

(e) For the Planck function expressed per unit frequency, what is the most probable frequency ν_p at a given temperature T? For the Planck function expressed per unit wavelength, what is the most probable wavelength λ_p at a given temperature T? For what range of temperatures does λ_p fall in the visible range of the electromagnetic spectrum?

By "most probable," we mean "where the spectrum peaks (is maximized)." This problem is really just an exercise in calculus and algebra. Your answer for $\lambda_{\rm p}$ should match the Wien displacement law given in standard textbooks. But given your results in parts (a)–(c), you should better appreciate why $\nu_{\rm p} \neq c/\lambda_{\rm p}$. And when someone shows you a spectrum in the future, you will appreciate why it is important to ask whether the spectrum is binned per wavelength or per frequency.