Astro 7A – Problem Set 4

1 Iron Fluorescence

Iron has an atomic number $Z = 26$. Consider the electron on its innermost shell. Calculate, using the Bohr model of a one-electron atom, the equivalent Lyman-alpha transition in iron (analogous to Lyman-alpha in hydrogen). Express the transition energy in electron-volts (eV). Do radio, optical, infrared, or X-ray astronomers detect this transition?

NB: The energy you calculate is near that of the "iron K-α fluorescence line", observed in astrophysical sources such as active galactic nuclei. Neutral or nearly neutral iron "fluoresces" every time an inner shell electron gets ejected—say by photoionization in the strongly irradiated gas surrounding a black hole—and an outer shell electron cascades down to fill back in the inner shell. We say a substance "fluoresces" when it absorbs high-energy radiation and emits lower-energy radiation as a consequence.

Even though this problem concerns strictly "hydrogen-like" Fe in which all 26-1 = 25 electrons are stripped away, it is still a good guide for neutral or nearly neutral iron, as long as we are concerned with the electron in the innermost shell—since the innermost electron practically sees the $Z = 26$ nucleus naked, "unshielded" by outer shell electrons.

2 Spinning Carbon Monoxide

The ubiquitous carbon monoxide molecule, CO, is used by astronomers to trace the temperature and density of cool molecular gas, in everything from galaxies to circumstellar disks.

The molecule can emit line radiation by virtue of its rotation. That is, in transitioning from a state of high rotation (high kinetic energy) to low rotation (low kinetic energy), the molecule emits a photon.

(a) Estimate the wavelength of the lowest energy, rotational transition in CO ($J = 1$ to $J = 0$, where J is the rotational quantum number, analogous to the principal quantum number n for electronic states). Do this by modeling CO as barbell spinning about its axis of greatest moment of inertia.[1] Assume that the CO bond length is one Bohr radius. Recognize that angular momentum comes quantized in units of $\hbar$, so that the $J = 1$ state has $\hbar$ more angular momentum than the $J = 0$ state. The $J = 0$ state has an angular momentum of zero, and an energy of zero. Is this transition detected by radio, optical, infrared, or X-ray astronomers?

[1] You will need to decide what axis the barbell is spinning about. Different axes will have different moments of inertia. Choose the axis that gives the greatest moment of inertia.

(b) Based on this calculation, would you expect rotational transitions from bigger, more complex molecules to lie at longer or shorter wavelengths?

3 Optical This and That

This problem gives some basic practice in learning various astronomical terms related to the extinction of light by matter.

Consider a gas composed of spherical particles each of mass m and radius s. The number density of such particles is n (particles per unit volume). When a photon hits a given particle, it is completely absorbed.

Consider a rectangular slab filled with such particles. The slab is infinite in the horizontal (x and y) directions, but has finite height H vertically (in the z direction).[2]

(a) What is the *cross-section* σ per particle, in terms of the variables given?

(b) What is the vertical *column density* N of the slab, in terms of the variables given?

(c) What is the *opacity* κ of the slab, in terms of the variables given?

(d) What is the *mass density* ρ (mass per unit volume) of the slab, in terms of the variables given?

(e) What is the vertical *optical depth* τ of the slab, in terms of the variables given?

(f) One side of the slab is illuminated by a uniform flux F_0. What is the *flux* F emerging from the other side of the slab?

(g) Now repeat (f) but imagine the same slab is ALSO filled with a second kind of gas particle, each of mass m_2 and radius s_2, and having number density n_2. This second kind of gas particle is also completely absorbing.

(h) Check out the picture of the solar corona[3] at http://antwrp.gsfc.nasa.gov/apod/ap090726.html.

The solar corona is a spherical halo of plasma (completely ionized hydrogen) surrounding the Sun, having temperatures of millions of degrees K. It is surprising to find the material in this outermost halo so hot, since the surface temperature of the Sun is a mere $\sim$6000 K, too cold to heat the corona by mere radiation (photons). The corona is thought to be heated instead by magnetic processes, still not well understood. Somehow, magnetic fields on the Sun annihilate to accelerate particles to high kinetic energies.

[2] Also called a plane-parallel atmosphere.
[3] "Corona" derives from the Latin for "crown."

Regardless of its mysterious origin, the corona is visible to the naked eye during total solar eclipses as a ghostly halo. It shines in white light thanks to Thomson scattering of solar radiation by free electrons in the corona. The free electrons in the corona intercept a fraction of the photons emitted by the Sun, diverting them into your line-of-sight. Recall that the Thomson cross section per electron is $\sigma_{\mathrm{T}} = 6.6 \times 10^{-25}\,\mathrm{cm}^2$. The protons, which are equal in number density to the electrons (the plasma is composed of ionized hydrogen and has zero net charge), do not scatter light as well as electrons do, because protons are much heavier than electrons and are thus less easily accelerated by photons. Take the proton cross-section for scattering light to be zero.

If the number density of the coronal plasma is $\sim 10^6$ particles (both electrons and protons in equal numbers) per cubic centimeter, and the rough size of the corona is the radius of the Sun ($\sim R_\odot$), estimate ρ, κ, and τ in cgs units. Thus finally estimate the fraction f of sunlight from the photosphere that is intercepted (scattered) by the corona (this determines how bright the corona is).

4 "Make Like a Tree, and ..."

If all the leaves of a tree fall to the ground, how thick is the layer of leaves of the ground? Give your answer in leaf thicknesses.[4] Your answer should be good to within a factor of 3.

What is the optical depth presented by the leafy tree to someone lying below the tree? Why does this answer make sense from the perspective of the vegetation? (Be the tree.)

The above two paragraphs are flip sides of the same question. You can try the second paragraph before the first, or vice versa.

Hint: Though you may wish to do a detailed calculation to solve this problem, there's no need to do so. Simply giving the answer and a short explanation why your answer is correct is sufficient. (But if you insist on giving a detailed calculation, that's fine, too.)

[4] The title of this problem was inspired by a certain movie.