Astro 7A – Problem Set 5

1 Fast vs. Slow

The "f-ratio" or "f-number" of a focussing mirror (or lens) is its focal length f divided by its diameter d. Call the f-ratio R. Following (somewhat bizarre and confusing) custom, "f/6" means $R = 6$ (this is just an example).

An imaging CCD detector is placed on the focal plane of the mirror (we say the detector is placed at prime focus). It is desired to image an extended—i.e., angularly resolved—source of emission, say an Hα nebula. The fact that the source is resolved means that it covers many CCD pixels. Suppose that for $f = f_0$ and $d = d_0$, a certain CCD pixel lying on the nebula collects an acceptably large number of photons $N = N_0$ within exposure time $T = T_0$.

(a) Suppose $f = f_0$ for the mirror is fixed but d is variable. If we demand that $N = N_0$ photons still be collected by the same pixel, how does the exposure time T depend on mirror diameter d? Write down a proportionality: $T \propto$ some function of d that you are to specify.

(b) Suppose $d = d_0$ is fixed but f is variable. Again demanding that $N = N_0$ photons be collected by the same pixel, how does T depend on f? Write down the proportionality.

(c) Explain why small R corresponds to "fast optics" while large R corresponds to "slow optics".

2 "Out, Vile Jelly"

On the back of the human eyeball is the retina, lying one focal length = 17 mm away from the pupil lens. Light is focussed onto the retina and collected by photosensitive cells called rods and cones. Rods are more sensitive than cones to the absolute amount of light, while cones are sensitive to color.[1]

According to the most excellent text on "Optics" by Hecht:

"Just about at the center of the retina is a small depression from 2.5 to 3 mm in diameter known as the yellow spot, or macula. There is a tiny rod-free region about 0.3mm in diameter at its center, the fovea centralis. (In comparison the image of the full Moon on the retina is about 0.2 mm in diameter.) Here the cones are thinner ... and more densely packed than anywhere else in the retina. Since the fovea provides the sharpest and most detailed information, the eyeball is continuously moving, so that light from the object of primary

[1] The title of this problem was inspired by a certain play.

interest falls on this region....Cones in the fovea are individually connected to nerve fibers [connected to the brain]."

In bright light, when the entrance pupil measures about 3 mm in diameter, the cones of the fovea centralis provide the highest resolution images capable of the human eye.

Given only the information above, and without looking up the answer, estimate the number of cones on the fovea centralis. Your answer should be good to within a factor of 3. Explain carefully your reasoning.

3 The Allen Telescope Array

When completed, the Allen Telescope Array (ATA), funded in part by Microsoft co-founder Paul Allen, will comprise 350 radio dishes laid out over an area roughly 1 km on a side (take the maximum distance between two dishes to be about 1 km). In addition to doing traditional astronomy, it will search for radio signals from extraterrestrial civilizations (www.seti.org/ata). It will operate at wavelengths between 3 and 60 cm.[2]

(a) Given N dishes where N is even, how many independent *two-antennae baselines* (distinct pairs of dishes) can be constructed? Express as a function of N, and evaluate for the ATA.

The signals from each distinct pair of dishes need to be combined electronically and processed by a computer to construct an interference map. The formula you are asked to compute is used routinely in deciding how powerful a computer is needed to process the data from a radio array.

(b) Approximately what will be the *highest* (i.e., best or finest) angular resolution achievable with the ATA? Express in arcseconds.

4 The Incredible Shrinking Shell

Consider a star surrounded by a spherical shell of hydrogen. The shell is pulled radially inward by the central star's gravity. We say the shell is *accreting* onto the star. The shell's radius is decreasing with time; the gas in the shell has a radial infall velocity. This situation occurs in *protostars*: young stars that are still gathering mass from the parent interstellar medium.

Take the star to be an emitter of continuum radiation (i.e., it radiates at all wavelengths).

[2] Neutral hydrogen emits prodigious amounts of 21 cm wavelength radiation, and 21 cm observations are a principal way by which we map hydrogen gas in galaxies.

Ignore any spectral lines the star may have intrinsically. Assume the star is stationary relative to the observer at Earth.

Consider the 3-2 Hα transition of hydrogen in the shell. A fraction of the hydrogen is in the $n = 3$ state. Another fraction is in the $n = 2$ state. Take the total density in the shell to be constant, and so low that optical depths in the shell are < 1 at all wavelengths and along all possible directions.

Assume that whatever intrinsic luminosity the shell has in Hα is much less than the continuum luminosity from the star at and near the Hα wavelength. In other words, the star is a lot brighter than the shell at wavelengths at and near Hα. Ignoring the shell, the continuum spectrum of the star near Hα is flat: F_λ is constant with λ.

An observer takes the spectrum of the entire system, in an angularly *unresolved* observation. By considering lines of sight marked A through D in Figure 1, *sketch* the $H\alpha$ spectral line profile. The axes of your plot should be flux density F_λ vs. wavelength λ. Aside from marking clearly where the rest wavelength λ_0 lies, your sketch need not be quantitative. Your sketch should, however, indicate clearly whether absorption and/or emission lines due to the shell are present. Consider carefully how the spectrum behaves both redward and blueward of λ_0.

You may find it useful to rank order the fluxes and wavelengths corresponding to the various sightlines. That is, arrange in increasing order $F_\lambda(A)$, $F_\lambda(B)$, $F_\lambda(C)$, and $F_\lambda(D)$. Repeat for $\lambda(A)$, $\lambda(B)$, $\lambda(C)$, and $\lambda(D)$.

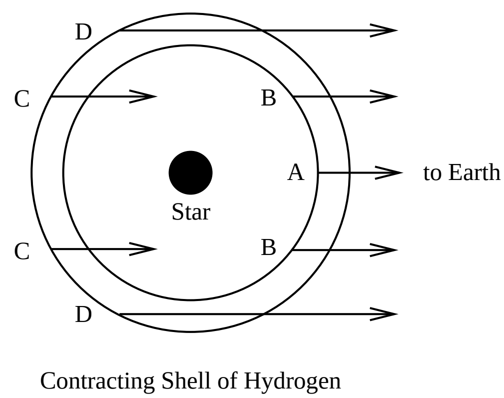

Figure 1: A shell of hydrogen infalling onto a central star. The radial infall velocity vectors are NOT shown. Arrows mark lines of sight A through D and all point towards the distant observer at Earth. All lines of sight contribute to the spectrum as measured by the observer, who is too far away to angularly resolve the source.