Astro 7A – Problem Set 6

1 Power Laws are Straight Lines in Log-Log Space

This problem demonstrates that *power laws*—some function $F(x) \propto x^\alpha$ for some α—are conveniently plotted in *log-log* coordinates. In such coordinates, power laws yield nice *straight lines*. The slope of the line equals α—a result that holds no matter what base logarithm you choose.[1]

Take the log of both sides of Kepler's Third Law

$$P = \frac{2\pi a^{3/2}}{\sqrt{G(m_1 + m_2)}} \tag{1}$$

to see that this is true. We say that Kepler's Third Law is a power law for the orbital period versus semimajor axis: a "power law of slope 3/2."

This problem verifies that Kepler's Third Law applies to the four moons that Galileo discovered orbiting Jupiter (the Galilean moons): Io, Europa, Ganymede, and Callisto.

You may neglect the masses of the moons compared to the mass of Jupiter.

(a) Using data for the orbital periods and orbital semimajor axes of the moons (you can look these up on the web or in books), create a graph of $\log_{10} P$ versus $\log_{10} a$.

(b) From your graph, show that the slope of the best-fit straight line through the data is pretty nearly 3/2.

(c) Calculate the mass of Jupiter from the value of the y-intercept.

2 Eccentric Comets

Cometary orbits usually have very large eccentricities, often approaching unity. Halley's comet has an orbital period of 76 years and an orbital eccentricity of $e = 0.9673$.

For this problem you may neglect the mass of the comet compared to the mass of the Sun.

(a) What is the semimajor axis of Comet Halley's orbit?

[1] If you are feeling perverse, you can even mix bases—say by taking $\log_e$ for the y-axis, and $\log_{10}$ for the x-axis—and you would still get a straight line of slope α. But the intercept does depend on the base, and would be screwy if you mixed bases.

(b) Use the orbital data of Comet Halley to estimate the mass of the Sun.

(c) Calculate the distance of Comet Halley from the Sun at periapse and apoapse.

(d) Determine the orbital speed of the comet when at periapse, at apoapse, and on the semiminor axis of the orbit.

3 Weighing Sagittarius A*

At the center of our Milky Way Galaxy, the motions of a number of stars have been tracked for several years. See the Quicktime .mov movie that is linked to the Astro 7A problem set webpage (http://astro.berkeley.edu/˜echiang/Astro7A/ps7A.html). This animation was constructed from real observational data collected by Andrea Ghez's UCLA Galactic Center Research Group. Aside from the fact that it was filmed at infrared wavelengths, using "adaptive optics" techniques that eliminate the blurring effects of the Earth's atmosphere,[2] it is simply a movie of some stars seen directly on the sky.

This problem takes a stab at estimating the mass of the object around which the stars are orbiting. That object, called Sagittarius A*, is marked by the star-shaped symbol in the movie. It is a supermassive black hole sitting at the center of our Galaxy.

(a) Take a close look at a few of the orbits. Why doesn't Sagittarius A* lie at the focus of every ellipse? Is something wrong with Kepler's First Law?

(b) Study the star marked S0-37. Note that Sagittarius A* does not lie at the focus of the ellipse traced by S0-37. Instead Sagittarius A* appears to lie close to the geometric center of the ellipse. Estimate the mass of Sagitarrius A* using S0-37. Describe your reasoning step-by-step. There's no need to be very precise or accurate, but you should be able to get to within a factor of 3 of the answer.

The distance to the Galactic Center is estimated to be 8 kpc (kiloparsec).

You may neglect the mass of the star compared to the mass of the supermassive black hole.

[2] We saw a movie of how this worked in class. Mirrors are continuously adjusted to flatten the wavefronts of stars (converting "curled potato chip wavefronts" distorted by the atmosphere into "flat pancake wavefronts").