Astrophysical Fluid Dynamics – Final Exam

Problem 1. Adiabatic Wind

A spherically symmetric wind blows steadily from a star. The wind behaves adiabatically with the usual adiabatic index γ. At infinite distance from the star, is the wind velocity finite or infinite? Justify your answer.

Problem 2. Bonnor-Ebert Mass

A cloud of gas having sound speed c_s is bounded by an external medium having pressure $P_{\rm ext}$. Derive an order-of-magnitude expression for the critical cloud mass above which the cloud collapses.

Problem 3. The Brünt in Disks

Consider a vertically thin, circumstellar disk in vertical hydrostatic equilibrium. Neglect disk self-gravity.

Estimate, to order-of-magnitude, the frequency of buoyant vertical oscillations in the disk, $\omega_{\rm B-V}$. Define whatever variables you need, but express your answer in terms of the local orbital angular frequency, Ω.

Problem 4. Sub-Keplerian Magnetized Disks

Consider a circumstellar disk threaded with a poloidal magnetic field, i.e., a field whose only components are in the radial and vertical directions. Work in the ideal MHD limit and neglect disk self-gravity.

The field geometry is such that magnetic tension forces the disk to rotate at a substantially sub-Keplerian rate. Its rotation velocity is, say, 0.5 that of the Kepler velocity because of magnetic tension.

Sketch the disk and its field. Estimate the plasma β (ratio of thermal pressure to magnetic pressure) required to enforce substantially sub-Keplerian rotation. Define whatever variables you need.

Problem 5. Instabilities

State as concisely but as completely as possible the requirements for the following instabilities:

1. Toomre instability in disks

2. Magneto-rotational instability
3. Thermal convection
4. Richardson shear turbulence

Credit awarded for quantitative statements.

Problem 6. Tabletop Dynamos

The magnetic diffusivity of liquid mercury is $7 \times 10^3\,\mathrm{cm}^2\,\mathrm{s}^{-1}$.

Explain quantitatively why it is so difficult to study magnetic dynamos using table-top experiments.