Order-of-Magnitude Physics – Problem Set 3

Due at the beginning of class.

Do any 1 of the problems + the last question (make up your own question).

You are free to do more if you like; answers will be graded.

Quote of the week:

> "Forefront research can begin (and end) on the back of a cocktail napkin. The utility of order-of-magnitude estimation extends through all phases of research. At a project's beginning, evaluating a few numbers can decide whether a speculation holds water or whether it should be abandoned. Later, in the thick of a complex experiment or calculation, order-of-magnitude estimation helps to diagnose and check. And at a project's close, order-of-magnitude physics provides the gift-wrap: the most compelling discoveries are not only supported by reams of data, but can also be communicated through a toy model or a few lines of boardwork." — Anonymous

Problem 1. Bright Lights, Big City

From John Harte's *Consider a Spherical Cow*:

How much warmer is a big city (say 10^7 people in a square 20 km on a side) than the surrounding countryside? Treat two cases:

(a) The city is trapped under a breezeless inversion layer, so all heat must be radiated.

(b) The heat is convected up into the atmosphere and carried away by horizontal winds.

Be sure to account for the fact that a human being today produces far more heat than just body heat.

Problem 2. Waterworld

(a) The Earth's atmosphere is projected to warm by about $\sim 3.5^\circ$ C over the next century due to increasing concentrations of carbon dioxide.

If the ocean also warms by this amount, by how much will the sea level rise worldwide? Neglect melting of ice sheets.

(b) Estimate the sea level rise due to the melting of Alaska's glaciers, which are projected to disappear within the next century.

(c) How much denser is the water at the bottom of the ocean compared to the top? Neglect variations in salinity. If this density contrast were to (magically) disappear, how much higher would the Earth's oceans be?

Problem 3. Water Striders

Based on research on bio-locomotion by Hu, Chan, and Bush (2003):

Water striders are insects that can walk on water. The picture on the website is that of the common water strider (*Gerris remigis*).

Consider instead the largest of water striders, *Gigantometra gigas*, having a reputed mass of 3 gm (not pictured). The shape of *Gigantometra gigas* is similar to that of the pictured strider insofar as it also balances principally on the longest 4 of its 6 legs. (The other two, shorter legs are for grabbing prey).

It is known that water striders are *not* isometric; that is, the relative proportions of their lengths (e.g., body length to leg length) do not remain constant as the absolute size of the insect increases. Therefore do not make the mistake of assuming that you can derive answers to questions below merely by scaling from the picture on the web. In any case, a scale bar for the picture on the web has not been provided.

Observe in the picture that all legs press *dimples* into the surface of the water.

(a) Each of the 4 long legs of *Gigantometra gigas* must be in a certain minimum contact with the water, as measured parallel to the leg. Estimate this minimum contact length, per leg.

(b) Estimate the characteristic length of a dimple, measured perpendicular to a long leg.

You may use a generalized version of Archimedes' principle: the mass of water displaced by an object floating in water, even if that object does not pierce the surface of the water, equals the mass of the object.[1]

(c) The pressure at the bottom of the dimple is greater than at the top. Are buoyancy (pressure) forces exerted by the water on the wetted bottoms of the legs important for support against gravity? Provide a quantitative assessment. Assume the cross-sectional radius of a leg is $w \approx 100\,\mu$m.

Problem 4. Ask Your Own Question

Ask an OOM question of your own. You don't have to answer it.

[1]Mansfield et al. (1997, Philosophical Transactions of the Royal Society of London, Series A, 355, 869); Keller (1998, Physics of Fluids, 10, 3009).