Order-of-Magnitude Physics – Problem Set 5

Due at the beginning of class.

Do any 1 of the problems + the last question (make up your own question).

You are free to do more if you like; answers will be graded.

"Just do it." — *Nike*

Problem 1. Planet Formation

Consider two identical spheres having radius s, elastic modulus M, and density ρ. They collide at relative velocity v.

When they collide, they deform: see Figure 1, which shows the moment of maximum deformation.

(a) Derive an order-of-magnitude formula for r, the radius of the "smooshed cap", in terms of the given variables. Note that $r \neq \Delta s$.

Hint: The (dimensionless) strain is the *gradient* of the displacement, not the displacement itself. There is more than one strain in this problem. Select the *maximum* gradient as that will dominate the stress.

(b) Solids have surface tension, too, because they have unsaturated bonds on the surface looking for partners to bind with. Laboratory measurements give a surface tension γ for silicate similar to that of water.

Derive an order-of-magnitude formula for the critical speed $v_{\rm crit}$ below which the spheres stick upon impact, and above which they bounce. Express your formula in terms of all the variables given.

Evaluate $v_{\rm crit}$ in cm/s for $s = 1\,\mu$m and $s = 1\,$m and thereby develop a sense for how far particle-particle sticking by chemical bonds can take you down the road to planet formation.

If you wish, you can check your formula against what Chokshi et al. (1993, Astrophysical Journal, v407, 806) found. Your answer should match in the scalings but is not expected to match in the numerical pre-factor.

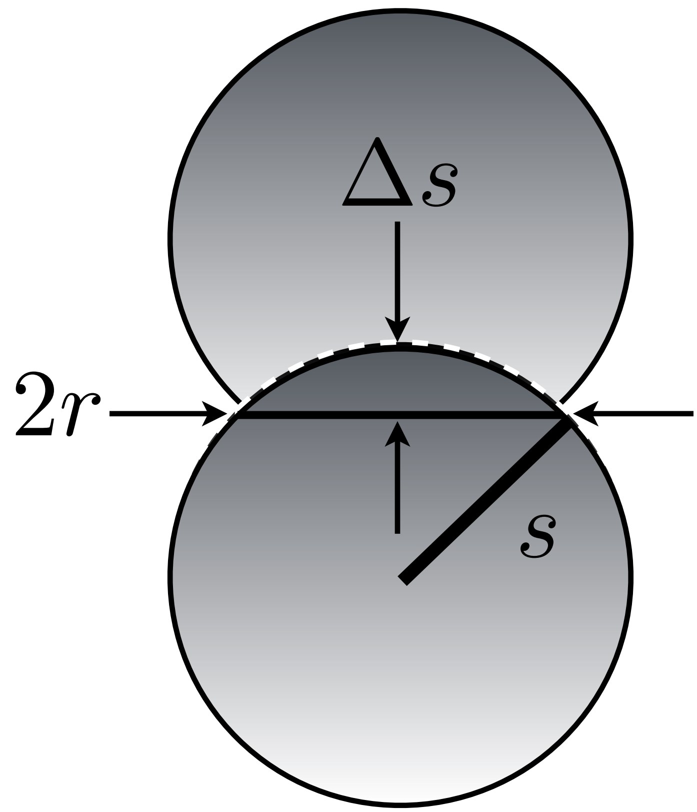

Figure 1: Two smooshed spheres. Notwithstanding the author's ineptitude with Keynote, both spheres are smooshed inward by Δs. The radius of the smooshed, flattened cap is r.

Problem 2. "If Frogs Can Fly ..."

"... then there is no reason why John Major cannot be Prime Minister." — Reaction of the British press after researchers at the Nijmegen High Field Magnet Laboratory in the Netherlands announced they had successfully levitated a 1-cm diameter baby frog in their Bitter magnet.

A Bitter magnet (after the physicist Francis Bitter) is essentially a solenoid where the usual coil of wire is replaced by vertically thin, radially wide plates stacked helically—like a continuous, tightly wound, spiral staircase. You can see pictures of Bitter plates in slideshows on http://www.magnet.fsu.edu/mediacenter/slideshows/index.html, created by the National High Magnetic Field Lab at Florida State University.

Bitter plates are punched with holes so that water can cool them when current flows through the plates. The bore hole (axis) of the solenoid is left empty for researchers' test subjects. In the case of Andre Geim's experiments, subjects have included water droplets, hazelnuts, mice, and pizza.[1]

(a) To acquire intuition about magnetic fields, consider the analogous and hopefully more familiar case of electric fields.

Consider two charges, e and $-e$, fixed to a rigid rod of length d. This electric dipole is placed in an electric field $\vec{E}(\vec{x})$ that varies with position $\vec{x}$. Assume that the field varies over a lengthscale $\gg d$.

Write down the net force $\vec{F}$ felt by the electric dipole. For simplicity you may orient the dipole so that it is parallel with the field. Your expression need only be good to order-of-magnitude.

Hint: The grad operator is your friend.

(b) By analogy, write down an expression for the net force $\vec{F}$ felt by a magnetic dipole of moment $\vec{\mu}$ immersed in a spatially varying field $\vec{B}(\vec{x})$.

(c) Model the Nijmegen magnet as follows. The plates are composed of copper, have an inner radius of $R_1 \approx 2\,\mathrm{cm}$, an outer radius $R_2 \approx 20\,\mathrm{cm}$, and are stacked such that there are $n \approx 10$ turns per cm. The length of the solenoid is $L \approx 30\,\mathrm{cm}$. The magnet is oriented so that its axis points parallel to Earth's gravity.

Decide where the frog can levitate (top, middle, or bottom of solenoid). Estimate the magnetic field required to levitate the frog. Give both a symbolic answer and a numerical estimate in Teslas.

(d) How much power is dissipated in the electromagnet? State in Watts.

(e) Imagine now a Bitter magnet large and powerful enough to levitate you. Repeat (c) and (d). Is human levitation practical?

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[1] In 2000, Andre Geim (of graphene and Scotch tape fame) was awarded the Ig Nobel Prize for his levitation experiments, sharing it with Sir Michael Berry (of Berry phase fame) who provided the underlying theory.

Problem 3. Watering Houseplants on Vacation

From Sterl Phinney:

A open-ended circular pipe of radius r is tilted at an angle of $45°$ to the vertical. Water is supplied at the top to replace what gravity pushes out the bottom.

For all parts, provide symbolic expressions in addition to numerical estimates.

(a) What is the maximum pipe radius $r_{\max}$ for which the flow will be laminar, not turbulent, assuming the flow is driven purely by gravity and not pressure? You can watch G.I. Taylor's "Low Reynolds Flow" video for guidance — assume a critical Reynolds number of 2000 for the transition between laminar and turbulent flow.

(b) To keep the pipe supplied with water, the top end is stuck into a hole in the bottom of a cup. Suppose the pipe has radius $r_{\max}$ and is $L = 5$ cm long.

There is a depth z_* of water in the cup below which the rate of outflow from the pipe does not depend on the depth in the cup, and above which the rate begins to increase significantly. Estimate z_*.

(c) How long would it take to empty the cup from depth z_*?

(d) What changes would you make to this device to water your houseplant while away on vacation?

Problem 4. Ask Your Own Question

Ask an OOM question of your own. You don't have to answer it.