Order-of-Magnitude Physics – Problem Set 6

Due at the beginning of class.

Do any 1 of the problems + the last question (make up your own question).

You are free to do more if you like; answers will be graded.

"Dare to be naïve." — Buckminster Fuller, architect and inventor
of the geodesic dome, after which carbon fullerenes are named.

Problem 1. Hot Springs

A popular form of relaxation therapy is immersion in a hot spring. Hot spring water temperatures are typically near 40°C (104°F). Assume for the problems below that the water is perfectly still if nobody is inside.

The thermal diffusivity of water is 6 times smaller than the momentum diffusivity of water. The ratio of $\nu/\kappa \approx 6$ is called the Prandtl number.

(a) How does the amount of heat that you absorb in a hot spring vary with the speed with which you wade? It is sufficient to provide a scaling. Assume that your wading speed is never zero.

(b) For a reasonable wading speed, how much heat (thermal energy) do you absorb per time? Express in Watts. Compare to your basal metabolic rate.

(c) After wading around for 10 minutes, you emerge and weigh yourself on a scale. How much weight have you lost? Express in lbs.

Problem 2. Centrifugation

Centrifugation is routinely used to separate cells from the media in which they are grown. The method relies on the density difference between cells (about $1.2\,\mathrm{g\,cm^{-3}}$) and media (about $1.0\,\mathrm{g\,cm^{-3}}$). Prior to being centrifuged, cells and media are well-mixed inside a flask. After centrifugation, cells are separated from media and can be collected in a dense pile at the bottom of the flask.

Estimate the rotation frequency (in rpm) of a centrifuge designed to separate cells from media in a 1 L flask. The cells are about $1\,\mu$m in radius and a routine lab task like this should be completed in 30 minutes.[1]

[1] According to the instructor's wife.

Problem 3. Epstein or Free Molecular Drag

A gas drag regime we did not treat in class but that is relevant in astrophysics is the Epstein[2] or free molecular limit, which obtains when (i) the mean free path for collisions between gas atoms is much longer than the obstacle size, and (ii) the relative bulk speed between the obstacle and the gas is much less than the gas thermal speed (i.e., the flow is subsonic). This regime applies to dust grains in gaseous flows.

Condition (i) implies that the gas does not behave as a fluid — instead, every gas atom that collides with the obstacle does so independently of every other colliding gas atom. In other words, after a gas atom ricochets off the obstacle, that atom essentially goes off to infinity and does not share its momentum with other gas atoms in the vicinity of the obstacle. Condition (i) is also called the "ballistic limit". It's every atom for itself!

(a) Consider a spherical obstacle of radius R moving at relative bulk speed u in a gas of mass density ρ and random thermal (a.k.a. sound) speed $c_{\rm s}$. By considering elastic collisions on both the front ("headwind") and back ("tailwind") sides of the obstacle, derive an order-of-magnitude expression for the Epstein drag force on the obstacle in terms of the variables given.

Use the fact that condition (ii) implies $u \ll c_{\rm s}$.

(b) Repeat (a) but for the case when condition (ii) does not hold, i.e., $u \gg c_{\rm s}$.

Problem 4. Ask Your Own Question

Ask an OOM question of your own. You don't have to answer it.

[2] Epstein, P., 1924, Physical Review, 23, 710, "On the Resistance Experienced by Spheres in their Motion through Gases". After you obtain your answer, you can check it by looking up this paper.