Astro 201 – Radiative Processes – Problem Set 10

Due in class.

Readings: RL 8.1 (yes, I know it was already assigned), 8.2; however much of RL 3.1–3.3 that you like.

Problem 1. Faraday Rotation

Consider the propagation of light through a magnetized plasma. The magnetic field is uniform $\vec{B_0} = B_0\hat{z}$. Light travels parallel to $\hat{z}$.

An electron in the plasma feels a force from the electromagnetic wave, and a force from the externally imposed B-field. Its equation of motion reads

$$m\dot{\vec{v}} = -e\vec{E} - \frac{e}{c}\vec{v} \times \vec{B_0} \tag{1}$$

where the electric field $\vec{E}$ can be decomposed into right-circularly-polarized (RCP) and left-circularly-polarized (LCP) waves:

$$\vec{E} = E_0(\hat{x} \mp i\hat{y})e^{i(k_{\mp}z-\omega t)} \tag{2}$$

where it is understood that the real part should be taken. The upper sign (-) corresponds to RCP waves, while the lower sign (+) corresponds to LCP waves. (And really, by this point in the class, do I really need to define all the variables?)

In the equation of motion above, we have neglected the Lorentz force from the wave's B-field, since it is small (by v/c) compared to the force from the wave's E-field.

(a) Prove that the solution of the equation of motion reads

$$\vec{v} = \frac{-ie}{m(\omega \pm \omega_{cyc})}\vec{E} \tag{3}$$

where $\omega_{cyc} = eB_0/mc$. This is the hardest part of the derivation for the dispersion relation of RCP and LCP waves.

(b) Rybicki & Lightman Problem 8.3

Problem 2. Everything That Has A Beginning Has An End

(a) Write down the electric field a distance r away from a monopole of charge q.

(b) Someone moves another monopole of charge $-q$ next to the original monopole. The two charges are separated by a distance b. Derive, to order-of-magnitude, the factor by which this dipole electric field is reduced from the monopole field.

(c) Someone moves two more charges, $-q$ and q, into position to form a square. The edge of the square has length b. Going around the square, the charges are $-q$, q, $-q$, and q. Derive, to order-of-magnitude, the factor by which this quadrupole electric field is reduced from the dipole field.

(d) Draw a picture of a pure electric octopole, that is, a charge distribution for which the electric field decreases as $1/r^5$ (and no less gradually). For bonus points, draw a picture of an electric hexadecapole (electric field dies no less gradually than $1/r^6$).

(e) Now imagine the dipole and quadrupole configurations in (b) and (c) rotating about their centers-of-charge with frequency ν. We have a rotating barbell and a rotating square, respectively. (Think CO and H_2).

To order-of-magnitude, what is the maximum distance from each object inside of which the electric fields are nearly perfectly in phase with the rotation?

This is the boundary of the *near zone*, inside of which the electric field geometry rotates with frequency ν as if it were a rigid body.

(f) Electromagnetic waves are emitted at the boundary of the near zone, into the far (radiation) zone.

Derive (one line of argument suffices) the factor by which the power carried by waves emitted by the rotating quadrupole is smaller than the power carried by waves emitted by the rotating dipole.