Astro 201 – Radiative Processes – Problem Set 12

Due in class.

Readings: RL 8.1, 8.2; RL 1.7, 1.8; Reader pages 203–212 (photocopied from Chamberlain & Hunten's textbook, "Theory of Planetary Atmospheres")

Problem 1. Faraday Rotation

Consider the propagation of light through a magnetized plasma. The magnetic field is uniform $\vec{B_0} = B_0\hat{z}$. Light travels parallel to $\hat{z}$.

An electron in the plasma feels a force from the electromagnetic wave, and a force from the externally imposed B-field. Its equation of motion reads

$$m\dot{\vec{v}} = -e\vec{E} - \frac{e}{c}\,\vec{v} \times \vec{B_0} \tag{1}$$

where the electric field $\vec{E}$ can be decomposed into right-circularly-polarized (RCP) and left-circularly-polarized (LCP) waves:

$$\vec{E} = E_0(\hat{x} \mp i\hat{y})e^{i(k_{\mp}z-\omega t)} \tag{2}$$

where it is understood that the real part should be taken. The upper sign (-) corresponds to RCP waves, while the lower sign (+) corresponds to LCP waves.

In the equation of motion above, we have neglected the Lorentz force from the wave's B-field, since it is small (by v/c) compared to the force from the wave's E-field.

(a) Prove that the solution of the equation of motion reads

$$\vec{v} = \frac{-ie}{m(\omega \pm \omega_{cyc})}\vec{E} \tag{3}$$

where $\omega_{cyc} = eB_0/mc$. This is the hardest part of the derivation for the dispersion relation of RCP and LCP waves. But it is fairly straightforward.

(b) Rybicki & Lightman Problem 8.3

OPTIONAL Problem 2. Combined Scattering and Absorption

Rybicki & Lightman Problem 1.10

Problem 3. Greenhouse Warming

Taken from Chamberlain & Hunten Problem 1.3

Assume that solar radiation is absorbed only at the Earth's surface where the albedo is 40%. The re-radiated energy is absorbed mainly by water vapor, which we approximate as a gray absorber with a density scale height of 2 km and total optical depth $\tau = 2$. Plot the temperature distribution with height for radiative equilibrium. What is the temperature discontinuity at the ground? What is the gradient in the air temperature near the ground, in K/km?