Astro 201 – Radiative Processes – Problem Set 2

Due in class.

Readings: Mihalas section 2-1.

Problem 1. Flat Disk SED

We return to the perfectly flat, blackbody disk encircling a blackbody star of problem set 1.

(a) Explain why νF_ν is a measure of the "flux radiated by an object per logarithmic interval in frequency." Here F_ν is the flux density [units of energy per time per area per frequency], and ν is the frequency of radiation. (Some refer more loosely to νF_ν as the flux radiated per octave in frequency. An octave, in either the acoustic or electromagnetic spectrum, represents a factor of 2 in frequency.)

Can you understand why νF_ν, and not F_ν, is a quantity of interest to those who wish to understand the overall energetics of an object? It is called the "broadband spectral energy distribution," or "broadband SED," or "SED" for short.

Hint: Most macroscopic objects in the universe are *broadband emitters*; that is, they radiate continuum radiation at all wavelengths. Plotting their spectrum would give a curve that varies smoothly with frequency. For any object, you can ask whether it is putting out its energy predominantly in X-rays, gamma rays, radio waves, infrared waves, etc (is it an "X-ray object?" "a gamma-ray object?" "an infrared object?") Now ask yourself why a person asking such questions should plot νF_ν and not F_ν.

(b) Is νF_ν equal to λF_λ, where λ is the wavelength of the radiation, and F_λ is per wavelength rather than per frequency? Show why or why not.

(c) Write down an expression for νF_ν for the blackbody disk. That is, write down the formula for the spectrum of the disk as measured by an observer for whom the disk is a point source.

Recall that the disk has a temperature $T(r)$ at every stellocentric radius, r. The inner radius of the disk is r_i and the outer radius is r_o. The disk is a distance D away from the observer, and is inclined by an angle i ($i = 0$ corresponds to a face-on disk). Your expression should take the form of an integral.

(d) OPTIONAL: *Sketch* (no heroics necessary, but get the orders of magnitude right and label the axes correctly) νF_ν vs. ν. If the spectrum exhibits power-law behavior, give the slope of the power law (i.e., $d\ln(\nu F_\nu)/d\ln\nu$). Overlay on your sketch the SED of the central stellar blackbody. Log-log space is best.

Problem 2. Practice with j_ν, α_ν, S_ν, B_ν, I_ν

(a) A plane-parallel slab of uniformly dense gas is known to be in LTE (local thermodynamic equilibrium) at a uniform temperature T. Its thickness normal to its surface is s. Its absorption coefficient is $\alpha_{\nu,\rm gas}$. Write down the specific intensity, I_ν, viewed normal to the slab, in terms of the variables given.

(b) The same slab is now filled uniformly with non-emissive dust having absorption coefficient $\alpha_{\nu,\rm dust}$. The dust is non-emissive, so its emissivity $j_{\nu,\rm dust} = 0$. Write down I_ν viewed normal to the slab, in terms of all variables given so far.

(c) The slab of gas and dust is further mixed with a third component: an emissive, non-absorptive uniform medium having emissivity $j_{\nu,\rm med}$ and absorption coefficient $\alpha_{\nu,\rm med} = 0$. Write down I_ν viewed normal to the slab, in terms of all variables given.[1]

OPTIONAL Problem 3. Photon Pinball in Clouds

NOTE: While this problem is optional, those students who have done it report back that they enjoyed it and found that the problem contained useful insights. Nevertheless, it covers material that is not traditionally part of a radiation course, and because of past criticism regarding excessive homework, I am leaving it optional.

This problem shows how the usual $e^{-\tau}$ attenuation factor for flux passing through an absorbing medium can be incorrect for flux passing through a purely scattering medium. (And thank goodness, or else it would be really dark on cloudy days.) Clouds can be viewed as a purely scattering medium.

Idealize the cloud as a uniform, 1-dimensional slab comprising particles that can only scatter light. A uniform flux of photons irradiates the top of the cloud deck (that is, just above the cloud deck, there exists a uniformly bright sheet). A photon passing through the cloud gets bounced like a pinball from cloud droplet to cloud droplet, preserving its frequency and never getting absorbed by any droplet. A few photons are lucky enough to make it through the cloud, while most get pinballed back out the way they came. We will calculate the fraction that make it through.

Take the cloud to have a droplet density [droplets per cubic volume] η, the droplet radius to be R, and the vertical thickness of the cloud to be z_{max}. Measure vertical distance through the cloud by z, where the top of the cloud is located at $z = 0$ and the base of the cloud is located at $z = z_{max}$.

[1] A physical realization of this problem might be an HII region surrounding an ionizing O star. The material in LTE would be the fully ionized plasma, emitting thermal bremsstrahlung radiation. The dust would be dust. The emissive, non-absorptive medium would be the same ionized plasma emitting recombination (line) radiation. For the assumptions stated in the problem to be valid, we would have to evaluate ν at, say, an optical recombination line like Hα.

(a) Write down the optical depth of the cloud, τ.

(b) *Incident* photons from the sun strike the top of the cloud. The photons have a number flux, F_i [number per time per area]. What is the *number density of incident photons* at the top of the cloud? Call this photon number density n_i. These incident photons have NOT been scattered yet by any droplet. Remember that flux is a number density multiplied by a speed.

(c) These photons random walk through the scattering droplets. Random walks are described by the diffusion equation,

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial z^2} \tag{1}$$

where D is the diffusion coefficient and n is the photon number density.

Express D in terms of symbols defined above and whatever fundamental constants you deem appropriate. Hint: dimensional analysis may prove useful.

(d) In steady-state, $\partial n/\partial t = 0$ (the number density of photons everywhere in the cloud does not change with time). Write down the solution to the diffusion equation for $n(z)$. You should have two, as yet unknown, constants of integration.

(e) To solve for the two constants of integration, you need two boundary conditions. The first condition is that $n_t = n(z = z_{max})$. Here n_t is the number density of photons at the base of the cloud. These photons comprise the *transmitted* flux.

The second condition is that the (net, number) flux, F, of photons at $z = 0$ equals the incident flux, F_i (directed down into the cloud) MINUS the outgoing, *reflected* flux, F_r (directed up, away from the cloud into space). Recall Fick's law, which is just another way of writing the diffusion equation, that the (net) flux $F = -D\partial n/\partial z$. So we have $F(z = 0) = F_i - F_r$.

Use the above, and the fact that the incident flux, F_i, must equal the reflected flux, F_r, PLUS the transmitted flux, F_t, to calculate T, the ratio of the transmitted flux to the incident flux, in terms of τ. Are you glad that clouds scatter but do not absorb light?

Problem 4. Galaxies

In an idealized model of a galaxy, stars are uniformly distributed within a cylinder of radius R and height H. The number density of stars is n [in units of stars per volume]. Model each star as a spherical blackbody of temperature T_* and radius R_*. The density of stars is so low that of all possible lines of sight running through the galaxy, the fraction that intersect a star is $\ll 1$.

An observer resolves the galaxy in a face-on viewing geometry, but does not resolve the individual stars making up the galaxy. In other words, the observer detects the galaxy as a circular disk that is very nearly uniformly bright from center to edge.

Write down the specific intensity the observer measures. Neglect terms of order H/d, where d is the distance to the galaxy.