Astro 201 – Radiative Processes – Problem Set 5

Due in class.

Readings: Course Reader: Bohren & Huffman pages 457–467, Draine review article. Rybicki & Lightman: Chapter 1 except the last section 8.1

Problem 1. Energy Density of Starlight and Grain Temperature

Interstellar space is filled with radiation. The bulk of the radiation arises from light emitted by the most massive (O-type) stars, each of mass $10^2 M_\odot$. The number of such stars in our Galaxy is about 5×10^4, distributed over a cylindrical disc of radius 50 kpc and height 200 pc.

(a) Estimate the energy density of starlight in the Galaxy. Express your answer in eV / cm^3. To estimate the luminosity of the most massive stars, use the fact that they are radiating at the Eddington limit; i.e., recall problem set 3.

(b) The interstellar radiation field (ISRF) heats dust grains in the interstellar medium (ISM). Estimate the temperature, T, of the largest grains in the ISM. These grains have radii of $a \sim 0.1$ μm. Take their emissivity (Q_{emis}) = absorptivity (Q_{abs}) to be unity for wavelengths shorter than $2\pi a$ and to fall off as $2\pi a/\lambda$ for longer wavelengths.

Is the grain hotter, cooler, or equal to the temperature of an ideal blackbody placed in the ISRF?

(c) At what wavelength, λ_{peak}, does νF_ν (the SED) of such grains peak? Give a number in microns, and also an expression in terms of T, fundamental constants, and dimensionless numbers.

(d) These largest grains also carry the lion's share of the mass in the interstellar grain distribution. Given a dust-to-gas mass density ratio of $\rho_{dust}/\rho_{gas} \sim 10^{-2}$ (metallicity), a rough average density of gas in the Galaxy of 0.1 H atom cm^{-3}, and a radial extent of the Galaxy of 50 kpc (kiloparsecs), calculate the specific intensity of the infrared Milky Way at a wavelength of λ_{peak}. Express in mJy arcsec^{-2}, where mJy = milliJansky = $10^{-3} \times 10^{-23}\,\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}}$ is a radio astronomer's unit of "flux density" (the "density" here refers to spectral density; i.e., it refers to the per Hertz) and arcsec = 1 arcsecond = 1/206265 (a phone number worth remembering) radians.

Problem 2. Dust Opacity

A rough model for the dust in the ISM tells us $dn/da \propto a^{-3.5}$, where dn is the differential number of dust grains having radii between a and $a + da$. The largest radius in the distribution is $a_{max} = 0.1\mu$m, and the smallest radius is $a_{min} = 0.001\mu$m.

(a) Plot the opacity, $\kappa(\lambda)$, contributed by all dust grains as a function of wavelength from $\lambda = 0.1\mu$m to $\lambda = 10\mu$m. Express the opacity in units of $\text{cm}^2\,\text{g}^{-1}$, where the g^{-1} equals "per gram of *gas*." Use whatever parameters you need as given by problem 1 of this set. *Consider only absorption and neglect scattering for simplicity.* At wavelengths much longer than a, the contribution to the opacity from scattering will be small compared to that from absorption.

Indicate over every decade in wavelength which grain sizes dominate the opacity. (For example, at wavelengths between $\lambda = 0.1$ and 1 μm, do the $a \sim 0.1\mu$m grains dominate the opacity? If not, do the $a \sim 0.01\mu$m grains dominate? And if not them, what about the $a \sim 0.001\mu$m grains?)

(b) Convert your plot to read *magnitudes* of absorption per kiloparsec travelled in the Galaxy (mag/kpc) as a function of λ. Again, use whatever parameters you need from problem 1 above.

A magnitude is a logarithmic unit measured in base 2.5; each magnitude of absorption reduces the flux from a star by a factor of 2.5; a rule of thumb is that 5 magnitudes is a factor of 100. (Editorial note: Astronomer's magnitudes were invented by Hipparcos to measure star brightnesses; a magnitude 0 star is 2.5 times brighter than a magnitude 1 star, and so on. For some reason, optical and near-infrared astronomers refuse to abandon this system, even though it runs backwards, crosses zero, and is normalized to different values depending on the wavelength band. Its only redeeming quality is that base 2.5 is close to the natural base e = 2.712...)