Astro 201 – Radiative Processes – Problem Set 6

Due in class.

Readings: Rybicki & Lightman 9.5, 10.6

Problem 1. Saha and Redshift of Recombination

Rybicki & Lightman problem 9.4, parts (a), (b), and (c)

(d) Calculate the redshift, z_{rec}, at which recombination occurred in the early universe. Use the temperature-redshift relation

$$T_{\text{photon}} = T_0(1 + z) \tag{1}$$

and the relation between baryonic number density and redshift

$$n = n_0(1 + z)^3 \,. \tag{2}$$

Here $T_0 = 2.73\,\text{K}$ is the temperature of the cosmic microwave photon gas today, and $n_0 = 10^{-7}\,\text{cm}^{-3}$ is the number density of baryons (read: hydrogen atoms) today, grossly averaged over the entire universe. You may define the epoch of recombination to be when 0.5 of the protons have recombined to form neutral hydrogen.

(Redshift is a cosmologist's measure of time. A redshift $z = 0$ corresponds to today. As one goes back in time, the redshift increases. Photons that have travelled from an epoch corresponding to a redshift z have their original wavelengths, λ, stretched to a longer wavelength, λ', by the expansion of the universe. By definition, $\lambda'/\lambda = 1 + z$.)

Problem 2. The Orion Nebula

(a) Sketch the flux density, F_ν, for the Orion Nebula due to its free-free emission from a wavelength of $\lambda = 100\,\mu\text{m}$ to $\lambda = 100\,\text{cm}$. Express F_ν in Janskys (10^{-23} is cgs units). Plot the spectrum on a log-log plot, and indicate power-law indices where appropriate.

Take the density of electrons to be $n_e = 2 \times 10^3\,\text{cm}^{-3}$, the electron and proton temperatures to be $T = T_e = T_p = 8 \times 10^3\,\text{K}$, the dimension of the ionized cloud to be $R = 1\,\text{pc}$, and the distance to the Orion Nebula to be $d = 500\,\text{pc}$. Take whatever geometry (cube, sphere) for the cloud is most convenient.

Please do not forget free-free self-absorption.

OPTIONAL (b) Overlay on your plot F_ν from pure electron-electron scatterings. Here aim only for order-of-magnitude accuracy. Remember that electron-electron collisions involve time-varying quadrupole moments, not time-varying dipole moments, and recall the quadrupole vs. dipole scalings from our discussion of Einstein A's.

For this part, consider the emissivity (j_ν) purely from electron-electron scatterings. Drop the emissivity from electron-proton scatterings. However, for the absorptivity (α_ν), consider the contributions from both electron-proton and electron-electron scatterings. If you think one absorption process is more important than the other, justify why you think that is the case and proceed by including just the one.

Problem 3. Blowing Stromgren Bubbles

Consider a lone O star emitting η Lyman limit photons per second. It sits inside hydrogen gas of infinite extent and of number density n. The star ionizes an HII region—a.k.a. a "Stromgren sphere," after Bengt Stromgren, who understood that such spheres have sharp boundaries—in which hydrogen is nearly completely ionized.

Every Lyman limit photon goes towards ionizing a neutral hydrogen atom. That is, every photon emitted by the star goes towards maintaining the Stromgren bubble. Put yet another way, no Lyman limit photon emitted by the star travels past the radius of the Stromgren sphere.

The rate at which Lyman limit photons are emitted by the central star equals the rate of radiative recombinations in the ionized gas. The sphere is *nearly* completely ionized.[1] These facts of photo-ionization equilibrium determine the approximate radius of the Stromgren sphere.

The temperature inside the sphere is about 10000 K. (A class on the interstellar medium can show you why.)

OPTIONAL (a) Derive a symbolic expression for the radius of the Stromgren sphere using the above variables and whatever variables were introduced in lecture.

(b) What is the timescale, t_{rec}, over which a free proton radiatively recombines in the sphere? That is, how long would a free proton have to wait before undergoing a radiative recombination? Give both a symbolic expression, and a numerical evaluation for $n = 1\,\mathrm{cm}^{-3}$.

OPTIONAL (c) If the star were initially "off," and the gas surrounding it initially neutral, what is the timescale for the Stromgren sphere to develop after the star were turned "on"? That is, how long does the star take to blow an ionized bubble? Think simply and to order-of-magnitude; you should get the same answer as (b).

[1] The sphere cannot be 100% ionized because then there would be no neutrals to absorb the Lyman limit photons that are continuously streaming out of the star.

Problem 4. Time to Relax in the Stromgren Sphere

It is often assumed that velocity distributions of particles are Maxwellian. The validity of this assumption rests on the ability of particles to collide elastically with one another and share their kinetic energy. For a Maxwellian to be appropriate, the timescale for a collision must be short compared to other timescales of interest. This problem tests these assumptions for the case of the Stromgren sphere of nearly completely ionized hydrogen of problem 3.

(a) Establishing the electron (kinetic) temperature: what is the timescale, t_e, for free electrons in the Stromgren sphere to collide with one another? Consider collisions occurring at relative velocities typical of those in an electron gas at temperature T_e. Work only to order-of-magnitude and express your answer in terms of n, T_e, and other fundamental constants.

I think it is fair to say that this problem is done most easily in cgs units.

(b) Establishing the proton (kinetic) temperature: repeat (a), but for protons, and consider collisions at relative velocities typical of those in a proton gas of temperature T_p. Call the proton relaxation time t_p.

(c) Establishing a common (kinetic) temperature: suppose that initially, $T_e > T_p$. What is the timescale over which electrons and protons equilibrate to a common kinetic temperature? This is *not* merely the timescale for a proton to collide with an electron. You must consider also the amount of energy exchanged between an electron and proton during each encounter. Estimate, to order-of-magnitude, the time it takes a cold proton to acquire the same kinetic energy as a hot electron. Call this time t_{ep}. Again, express your answer symbolically.

Hint: you might find it helpful to switch the charge on the electron and consider head-on collisions between the positive electron and positive proton.

(d) Numerically evaluate t_e/t_{rec}, t_p/t_{rec}, and t_{ep}/t_{rec}, for $T_e \sim T_p \sim 10^4$ K (but $T_e \neq T_p$ so that $t_{ep} \neq 0$). Is assuming a Maxwellian distribution of velocities at a common temperature for both electrons and ions a good approximation in Stromgren spheres?