Astro 201 – Radiative Processes – Problem Set 9

Due in class.

Readings: Rybicki & Lightman 4.1, 4.8, 4.9, 6.1–6.3, 6.6, skim 6.4–6.5, skim 6.8; copy of Shu's chapter on the "simple version" of synchrotron theory; however much of the classic and readable paper by Scheuer and Williams

NOTE FOR ALL PROBLEMS: In this class, we are not going to stress over the $\sin\alpha$ term in any expression involving synchrotron emission, nor are we going to worry about other factors of order unity, like gamma (Γ) functions. If you see a $\sin\alpha$ or gamma function in your travels, just set it equal to 1.

Problem 1. Synchrotron Losses

(a) Obtain an analytic expression for the energy of a single relativistic electron as a function of time, $E(t)$, taking into account its energy loss by synchrotron radiation. Your expression should contain only the variables $E(0)$ (the initial energy of the electron), B (the magnetic field, here held fixed with time, following the rest of the world, though one should worry in general about the field changing with time just as the electron energy spectrum changes with time), time t, and fundamental constants. Assume $\sin\alpha = 1$ (the electron pitch angle is 90 degrees) for simplicity.

For (synchrotron) problems of interest to us, the electron always remains relativistic. It merely evolves from a large $\gamma \gg 1$ to a smaller $\gamma \gg 1$.

(b) How can you reconcile the loss of energy of the electron with the bald statement of Rybicki & Lightman on page 168 that "γ is constant"?

OPTIONAL (c) We have made arguments in class that power-law distributions of electrons in astrophysical sources are maintained against synchrotron losses by continuous energization by central engines (a.k.a. *injection*). The injection (input) spectrum of electrons is modified by synchrotron losses to produce a steady-state (output) distribution.

Call $\eta(E,t) = dN/dE$ the differential energy spectrum of electrons as discussed repeatedly in class. Continuity of electrons in energy space reads

$$\frac{\partial \eta(E,t)}{\partial t} + \frac{\partial}{\partial E}[\dot{E}\eta(E,t)] = I \tag{1}$$

where I is the rate of injection of electrons with some input distribution and $\dot{E}$ is the rate of energy loss of a single electron by synchrotron radiation. This equation should not mystify you; it merely describes how the number of electrons in a given energy bin changes with time, taking into account a flux divergence (the second term on the left-hand-side) and a source term (the right-hand-side).

We have assumed in class a *steady-state* distribution of electron energies for which $\eta \propto E^p$. Given p, how must I scale with E? Give only the scaling and forget about the numerical coefficients.

As with most scaling problems, you don't have to solve anything in detail. Ruthlessly work to order of magnitude.

OPTIONAL (d) Electrons having a given energy must wait a characteristic time before synchrotron losses become important. Before this time elapses for all such electrons, how does η scale with E?

OPTIONAL (e) The *spectral* index ($\alpha = d\ln F_\nu / d\ln \nu$) of radiation in a fixed frequency range from a radio jet flattens with increasing distance from the central galaxy. That is, $\alpha = -0.5$ at the remote edge of the jet (the "hot spot"), and $\alpha = -1$ closer in. Given your understanding in (c) and (d), where are the "freshest" electrons located, i.e., those newly injected into the energy spectrum? Are they at the end of the jet, or are they closer in? In other words, where is the principal site of particle acceleration?

OPTIONAL (f) Sketch several profiles of η vs. E at various times, assuming I is constant in time.

Problem 2. Great Balls of Relativistic Fire

Rybicki & Lightman Problem 4.1 (Don't worry if you can't reproduce the answer to within a factor of 2; the answer is meant to be a rule of thumb.)

Problem 3. Superluminal Motion, or Photons Chasing Photons

Rybicki & Lightman Problem 4.7

(c) Plot v_{app}/c vs. θ for $\gamma = 10^2$. Does the viewing angle θ need to be especially small for superluminal motion to be perceived?

For culture: see the beautiful illustration of superluminal motion in the optical M87 jet by Biretta in the accompanying .jpg on the class website.