Astro 201 – Radiative Processes – Problem Set 10

Due in class.

Readings: RL chapter 4.1, 4.9; paper by Carilli et al. on class website; however much you like of the classic paper by Scheuer and Williams also on website.

NOTE FOR ALL PROBLEMS: In this class, we are not going to stress over the $\sin\alpha$ term in any expression involving synchrotron emission, nor are we going to worry about other factors of order unity, like gamma (Γ) functions. If you see a $\sin\alpha$ or gamma function in your travels, just set it equal to 1.

Work in the same spirit that we have been working throughout this class: order-of-magnitude. At the same time, try to hang onto the 4π's.

Problem 1. Leaving on a Jet Frame[1]

Two-sided jets travelling at close to the speed of light often appear brighter on one end than the other.

Consider a bipolar (two-sided), symmetric jet of material moving at BULK velocity v and having BULK Lorentz factor γ. The jet axis points at an angle θ relative to the Earth. One side of the jet is directed towards us, and the other side is directed away.

The jet is composed of electrons (possibly also positrons; the situation is unclear) that radiate synchrotron emission. In the rest frame of the jet, the electrons are still gyrating relativistically, and in that same rest frame, they emit a specific intensity $I_\nu \propto \nu^\alpha$ between frequencies ν_1 and ν_2. For this problem, take $\alpha < 0$.

Recall from the reading that I_ν/ν^3 is a relativistic invariant.

(a) Sketch the specific intensity versus frequency of the jet that points toward an observer on Earth, as seen by that observer.

Overlay on this sketch the specific intensity of the counter-jet (again, as seen by the Earth-bound observer).

Overlay also the specific intensity of either jet or counter-jet as measured in its rest frame. Compare in particular the rest-frame jet spectrum with the counter-jet spectrum observed at Earth.

Annotate your sketch with scales and indices, expressed symbolically in terms of the variables given above.

[1]Former BAD grad Julia Kregenow named this problem.

(b) By what (symbolic) factor is the forward jet brighter than the counter-jet in specific intensity at a fixed frequency as measured on Earth?

(c) The dynamic contrast in optical surface brightness between jet and counter-jet in M87 exceeds 450. If $\alpha = -1/2$, what are the constraints on γ, v, and θ?

Problem 2. In Space, No One Can Hear You Scream

Take a look at Figure 3, Table 3, and Figure 4 of Carilli et al.'s paper on the radio galaxy Cygnus A.

In Figures 4a and 4b, we can distinguish a break in the spectral index at about $\nu \approx 10^4$ MHz. The break is interpreted to represent the onset of synchrotron losses, as we studied in a previous problem. The corresponding electron energy spectrum will also be broken.

In either regime, provided we don't look at frequencies too low (lower than $\nu \approx 10^{2.75}$ MHz), the emission arises from optically thin material.

In Figure 3, one arcsecond equals one kpc.

Part (d) is independent of the other parts.

(a) Estimate the minimum magnetic field strength in "Hot Spot A" using Burbidge's minimum energy argument and using Figure 3, Table 3, and/or Figure 4, or any other raw data from Carilli et al.'s paper. Express in Gauss.

You will have to decide whether to use the spectrum below the break, above the break, or both. We want a *minimum* estimate of the magnetic field.

Interpret the unit of "(beam)$^{-1}$" (per beam) to mean $[\pi(2.25\,\mathrm{arcseconds})^2]^{-1}$. This is the angular area of the radio interferometer's beam, or footprint on the sky. When Carilli et al. cite "4.5 arcseconds resolution," I interpret the 4.5 arcseconds to be equal to the FWHM (full-width-half-maximum) of their beam.

(b) Estimate the minimum total pressure—magnetic plus material (electron)—in hot spot A. Express in cgs units. Compare to the pressure of air around you.

(c) Estimate the total energy (magnetic plus material) in hot spot A in ergs.

OPTIONAL (d) Estimate an age of the hot spot given the observation of the spectral break. Compare to the dynamical time of the jet (the time it took the jet to grow to its present size).

Problem 3. Galactic Synchrotron Emission

The *brightness temperature* of synchrotron emission in the Galactic plane is measured to be

$$T_b = 250 \left(\frac{\nu}{480\text{MHz}} \right)^{-2.8} \text{K}\,, \tag{1}$$

valid for 8 GHz $> \nu >$ 480 MHz. This emission arises from cosmic ray electrons gyrating in the Galactic magnetic field of strength $B \sim 3\mu$G. Take the size of the emitting region to be $\sim$10 kpc.

(Aside from being a nuisance foreground for CMB researchers, Galactic synchrotron emission is thought to probe the supernova rate in the Galaxy, since cosmic ray electrons and protons at the relevant energies are thought to be accelerated in supernova shock waves. In addition, a famous "FIR-radio correlation" is observed to exist for normal spiral galaxies; the radio traces electrons accelerated by supernovae, while the far-infrared (FIR) emission traces warm dust grains. Where there are supernovae, there is active star formation; where there is star formation, there are dust grains warmed by the ISRF. Or so goes the traditional interpretation of the FIR-radio correlation.)

(a) Provide an approximate expression for the differential energy spectrum of cosmic ray electrons. Express in units of particles $\text{m}^{-2}\,\text{sr}^{-1}\,\text{s}^{-1}\,\text{GeV}^{-1}$. Indicate the approximate range of energies (E_{min} and E_{max}) for which your expression is valid.

OPTIONAL (b) Compare your answer to the differential energy spectrum of cosmic ray protons, as given in the Reader. The units of the plot were accidentally chopped off; they are the same as those specified in part (a). By what factor do cosmic ray protons outnumber cosmic ray electrons?

(c) Estimate the gyro-radii of cosmic ray electrons and of cosmic ray protons at E_{min} and E_{max}. Compare to the size of the (baryonic disk of the) Galaxy, 10 kpc. From this comparison you should be able to understand why cosmic ray astronomy is so difficult.

OPTIONAL (d) Why do we ignore the contribution from cosmic ray protons to the Galactic synchrotron emission at the above frequencies ν? Provide a quantitative answer.