Astro 201 – Radiative Processes – Problem Set 9

Due in class.

Readings: Article by Kellerman; Rybicki & Lightman 7.1, 7.2, 7.4, as much as you like of 7.6, 7.7; however much of Blandford's lecture notes but watch for errors.

I did not think it needed to be said, but based on what some of you have confessed, I was wrong. Do not look up the answers to the RL problems in the back of the book. You are defeating the purpose of doing problems.

Problem 1. Compton Catastrophe

We derived in class that for a synchrotron-self-Compton (SSC) emitting source that is static in bulk, the ratio of the luminosity due to first-generation inverse Compton scattering of synchrotron photons to the luminosity due to synchrotron processes is

$$f \equiv \frac{L_{IC,1}}{L_{sync}} = C\nu_m T_{bm}^5 \tag{1}$$

where ν_m is the frequency at which the synchrotron spectrum peaks, and T_{bm} is the brightness temperature of the synchrotron radiation at that frequency.

(a) Derive an estimate for the constant C in terms of fundamental constants. Assume whatever geometry you like for the source. I found I had to use the (usual) constants e, k, c, and m_e. [Try not to let the "k" throw you; there's no temperature in the strict thermodynamic (i.e., Maxwellian) sense, but that doesn't stop the astronomers from making up a "brightness" temperature.]

You might find the heuristic derivation of the optically thick portion of synchrotron spectra helpful (very indirectly).

(b) If $\nu_m \sim 1\,\mathrm{GHz}$ (as it typically is for compact radio sources), what is the maximum T_{bm} above which $f > 1$? Express in Kelvin.

(c) Given f, what is $L_{IC,2}/L_{IC,1}$? That is, what is the ratio of second-generation to first-generation scattered power? Explain your answer.

(d) Consider the situation after N scatterings, where $N \to \infty$. What is the critical f for which the total luminosity of the source becomes unbounded? Careful estimates will be rewarded.

Problem 2. Compton Saturation

Rybicki & Lightman problem 7.1b (only b! We answered the other parts in lecture.)

Problem 3. Cold Plasma

Consider a non-relativistic electron moving at speed $v \ll c$. A single photon of original energy ϵ Compton scatters off the electron and suffers an energy shift of $\Delta\epsilon$. Work in the same Thomson limit that we have been working in throughout lecture.

(a) Estimate a typical value for the fractional change in photon energy, $\Delta\epsilon/\epsilon$. Express your answer symbolically. By "typical," we mean any old photon that comes in from any old direction.

(b) Consider this same electron flying through a sea of photons all of energy ϵ. Many photons get inverse Compton scattered to and fro.

Averaged over all the photons that got scattered, what is the mean fractional change in photon energy, $\langle\Delta\epsilon/\epsilon\rangle$? You may use our expression for the inverse Compton power scattered by a single electron if you wish. Neglect the change in the electron's velocity as it is bombarded by photons.

(c) Explain physically why parts (a) and (b) give different answers. (And then read Blandford page 175, if you wish. It is better to think about this problem and achieve your own understanding than to head straight for Blandford right away).