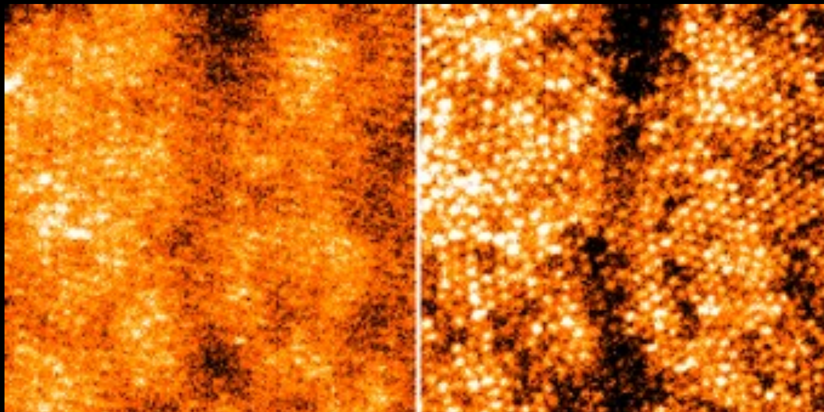
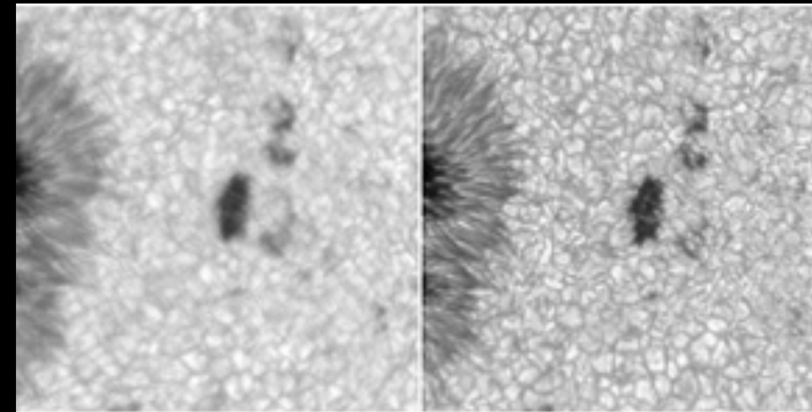
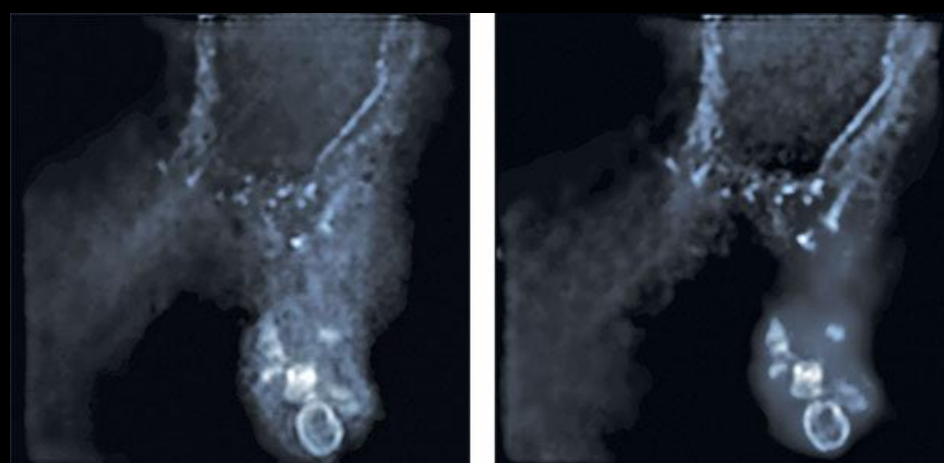




Adaptive Optics

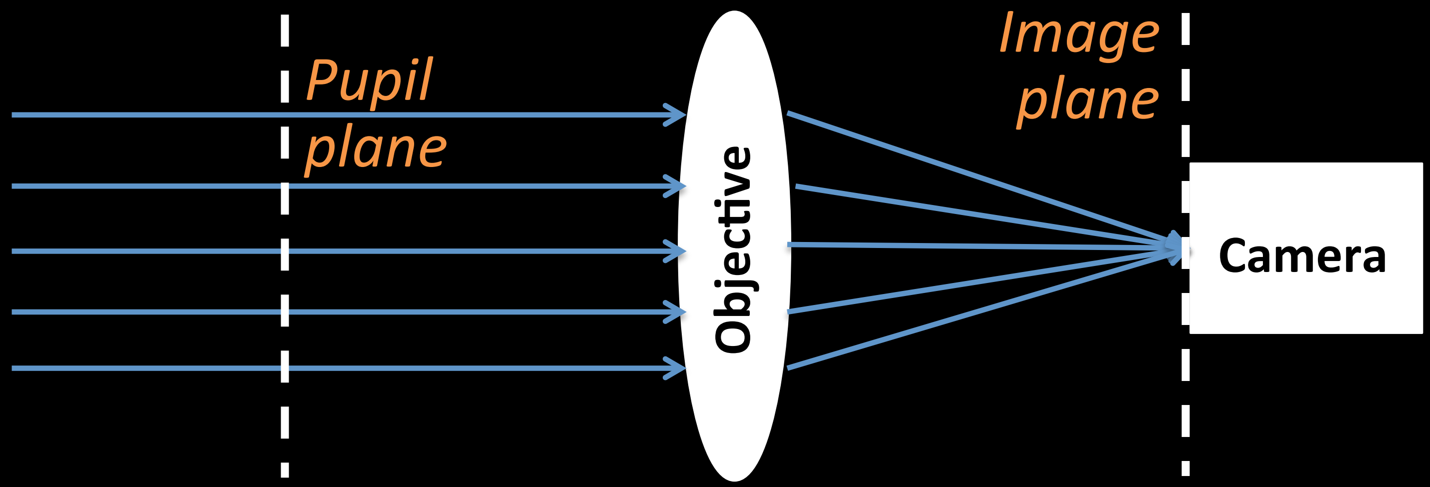
Special Topic in Astrophysics

ASTRON 250 - Fall 2013



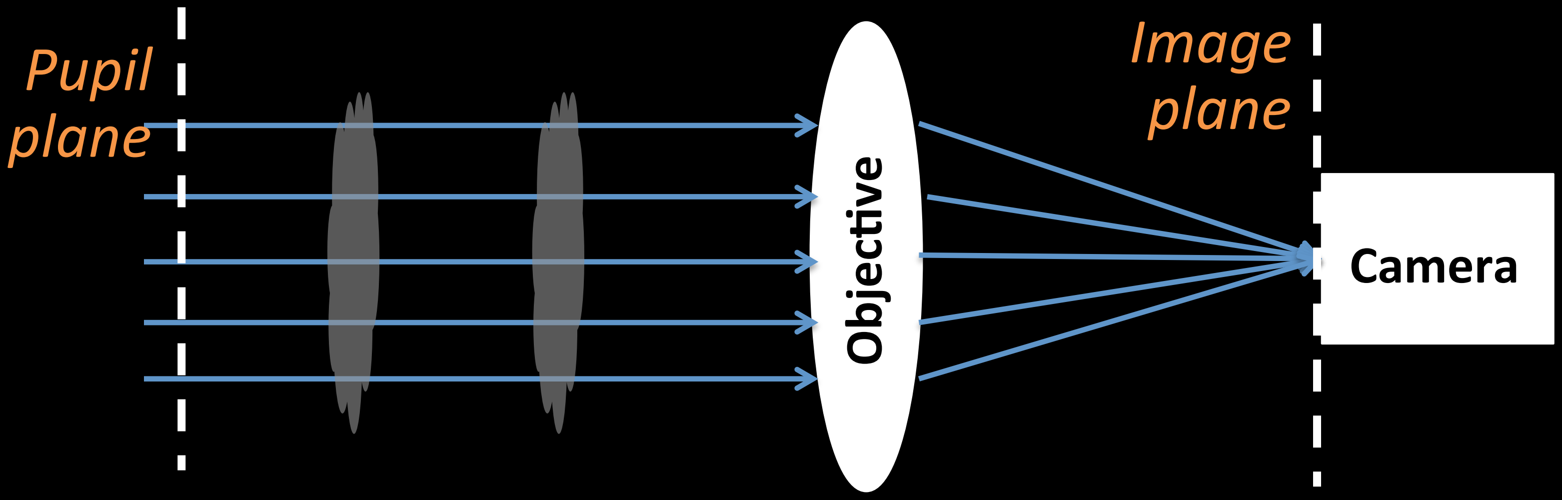
Optical conjugation

- The relationship between the E/B fields in the pupil and image planes is a Fourier transform
- Pupil and image planes are conjugated



Decomposing the OTF

- OTF = Π OTF only true if all turbulence phenomena are located in the pupil plane and in the case of small deviations
  - Referred to as the "near field" approximation
- Otherwise, take into account wave propagation



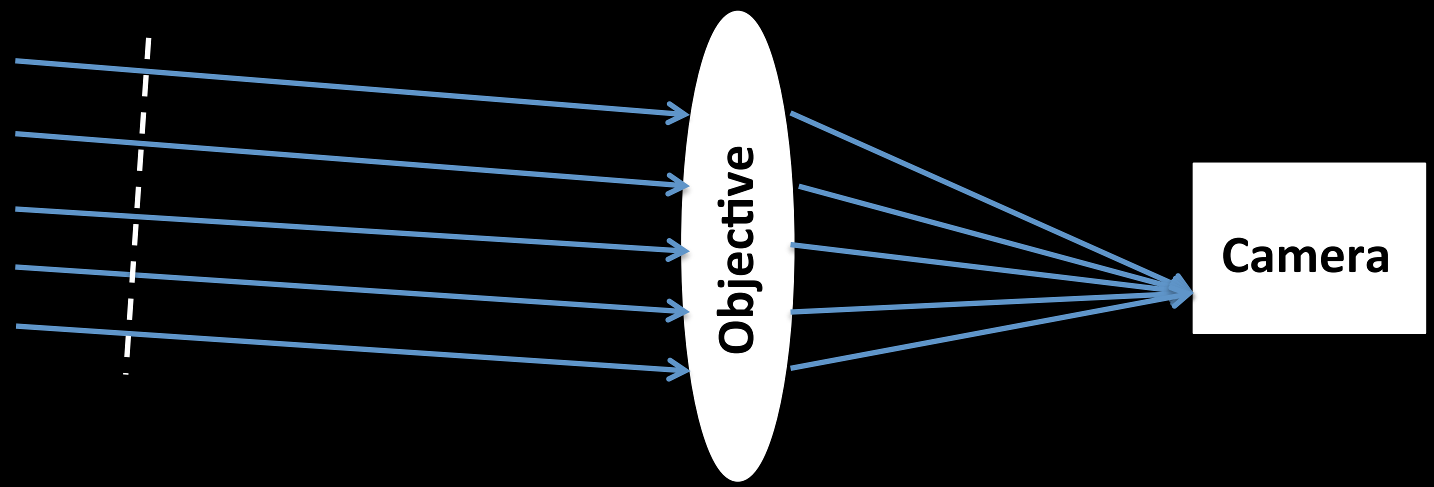
What type of aberrations?

Some simple aberrations

- Defocus
- Chromatic aberration
- Spherical aberration
- Astigmatism, coma
- …
- Static aberrations can be predicted and corrected for with corrective optics

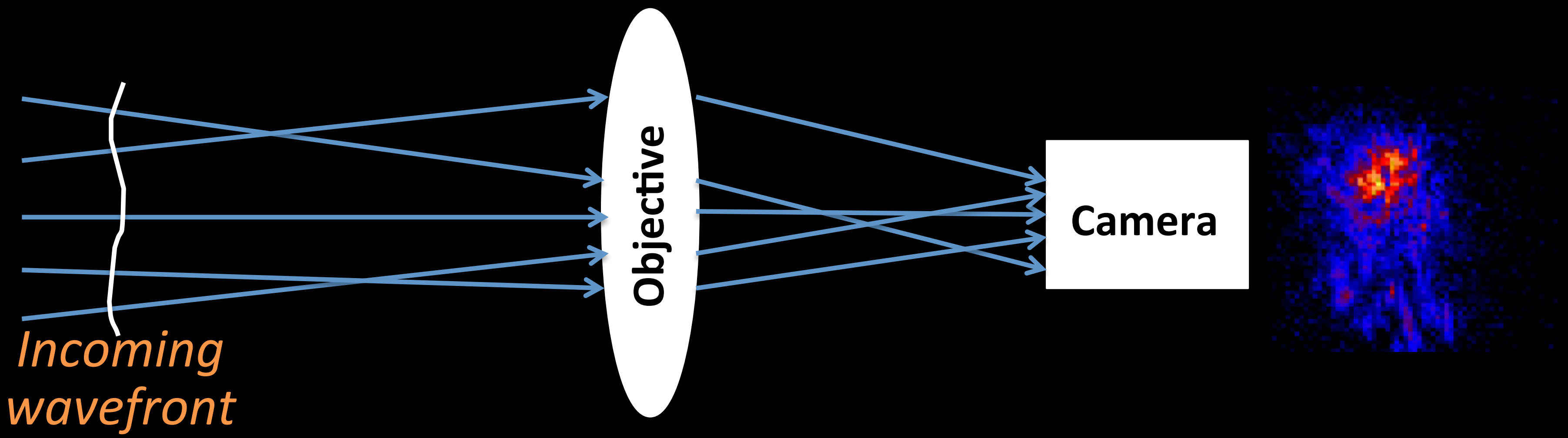
A particularly simple aberration

- Slanted incoming wavefront (tilt)
  - Phase in the pupil plane has a linear gradient
  - Results in a linear displacement in image plane
  - Image is still diffraction limited



More complicated aberrations

- In the general case, wavefronts are corrugated, and each little patch has its own local slope
- Can lead to image blurring or many displaced diffraction-limited spots ("speckles"), depending on amplitude of perturbations



Are there “tolerable” perturbations?

- Small enough phase differences do not preclude constructive interference, although it may reduce the resulting intensity
- Larger phase differences are destructive to the signal at the spatial frequency considered
- Phases should remain within π/2 of each other to ensure good coherence
  - This is Maréchal’s criterion

Describing a corrugated wavefront

- Any correction relies on the ability to <u>quantify</u> wavefront shape
- Two main options:
  - A discrete map of phase delays (and/or slopes) in small patches
  - A decomposition in a series of analytical functions that should be selected to be
    - orthogonal to one another (to ensure unicity)
    - “smart” (to minimize the number of terms)

General treatment of aberrations

- A standard method: decomposition in Zernike polynomials:

Radial term *Azimuthal term*

$$Z_n^m(\rho,\varphi) = R_n^m(\rho)\,\cos(m\,\varphi)$$ "even" terms

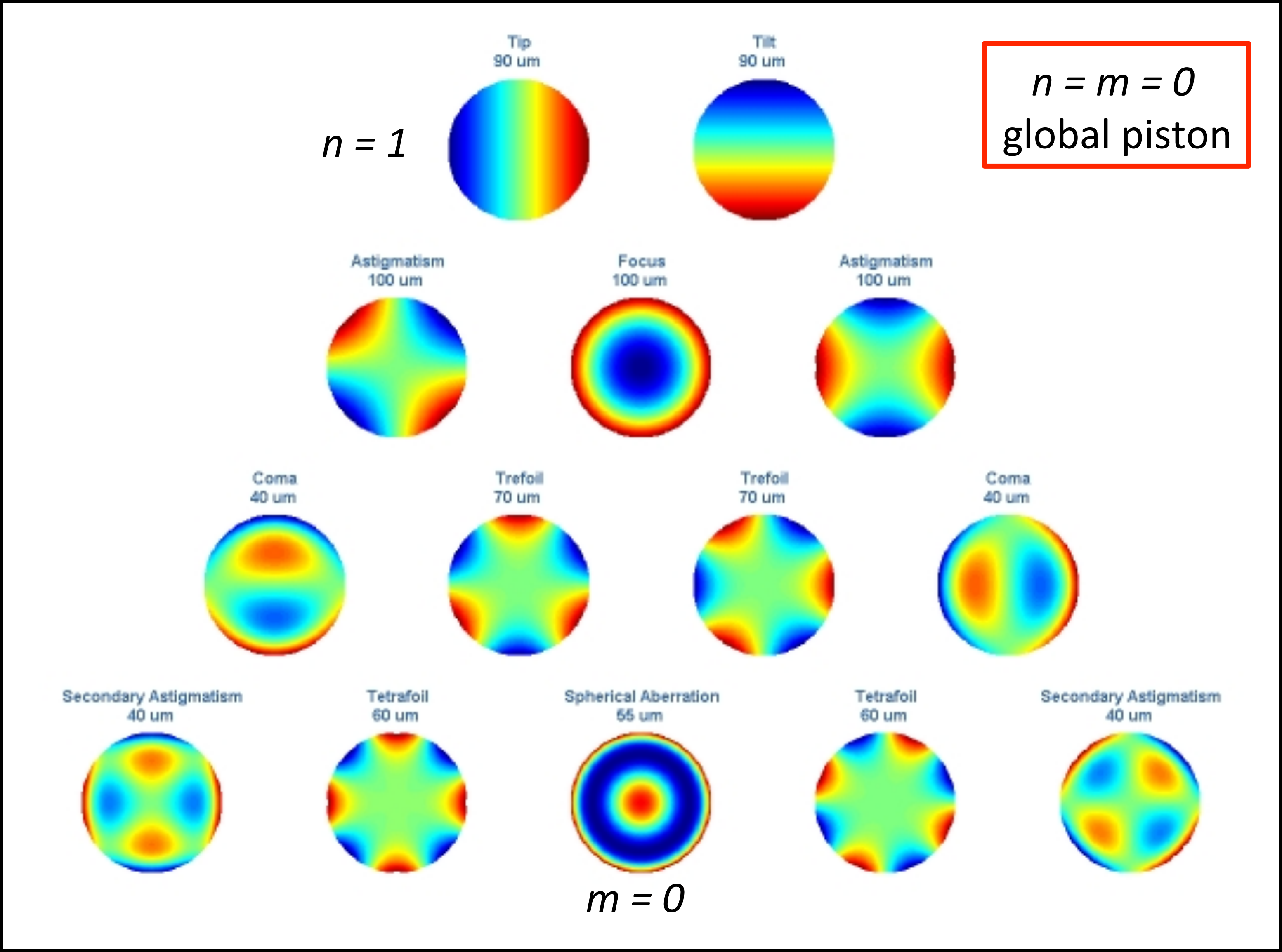
$$Z_n^{-m}(\rho,\varphi) = R_n^m(\rho)\,\sin(m\,\varphi),$$ "odd" terms

($n \geq m$)

$$R_n^m(\rho) = \sum_{k=0}^{(n-m)/2} \frac{(-1)^k\,(n-k)!}{k!\,((n+m)/2-k)!\,((n-m)/2-k)!}\,\rho^{n-2\,k}$$

if n-m is even, otherwise $R_n^m = 0$

A natural basis to describe aberrations



Know your enemy!

- One needs to understand (and measure) the properties of aberrations to correct for them, or anticipate their effects on the image
  - Ideally precisely (for static aberrations)
  - If they are randomly variable, key statistical properties and power spectrum can be enough

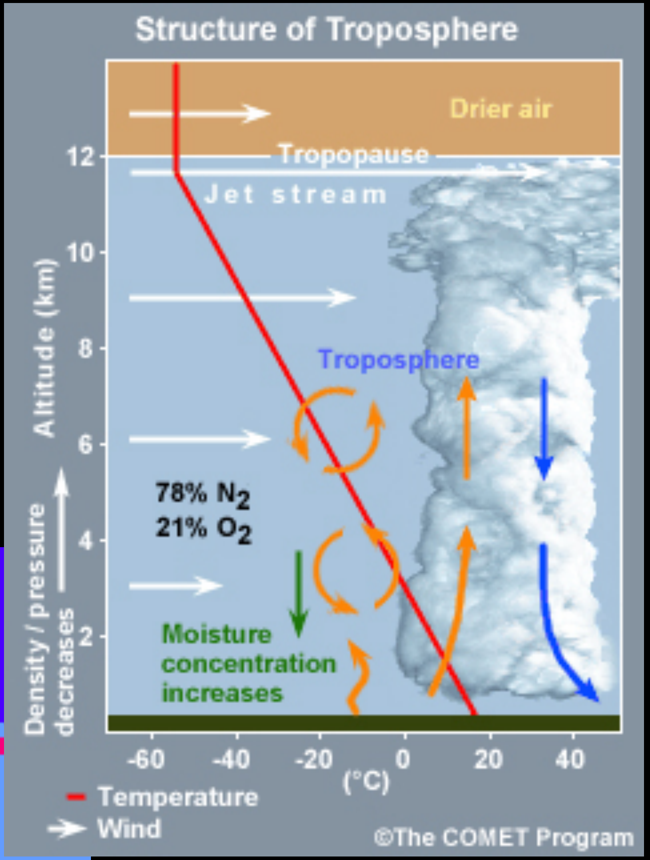
Observing through atmospheric turbulence

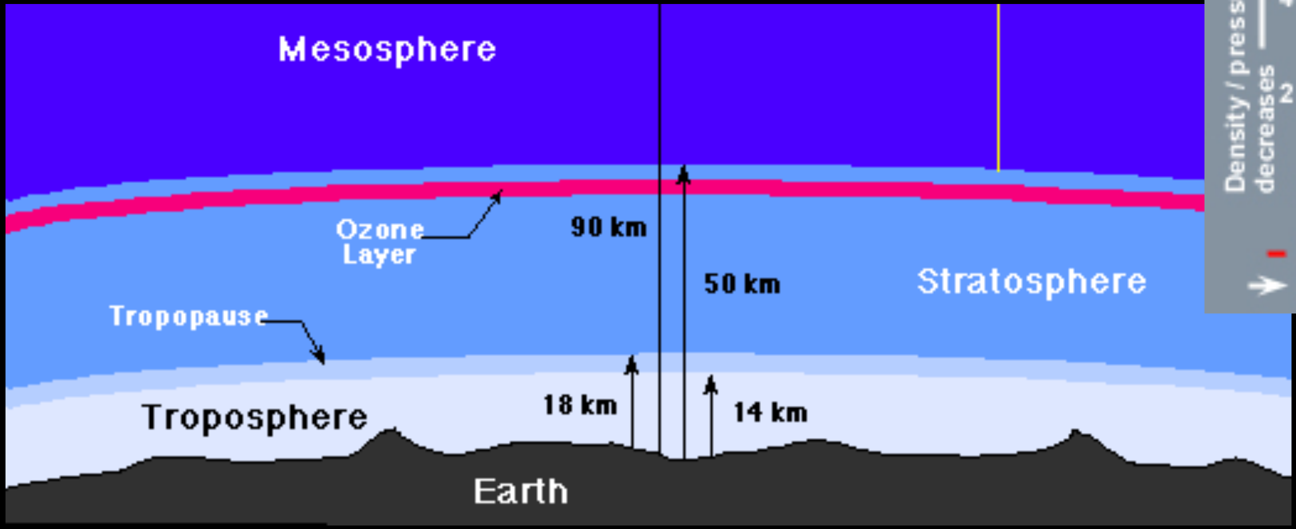
- Roddier readings

Observing through atmospheric turbulence

The Earth's atmosphere is characterized by fluctuations of its index of refraction due to variations in water vapor content.

Complex motions are also induced by winds (wind shears) and temperature gradients.





Problem set-up

- Atmosphere is constantly changing, so only a statistical picture can be drawn
- Atmosphere is described as a number of independent horizontal layers whose effects can be added
- In the frozen flow approximation, each layer is assumed to be a "phase screen" moving at a constant (wind) velocity

Kolmogorov turbulence

- Assumes an homogeneous atmosphere and isotropic, multi-scale turbulence
- Energy is injected at large scales and transferred to increasingly small scales
  - Inner scale: viscous dissipation (~ mm-cm)
  - Outer scale: the largest eddies (~ 10 m)
- Most of the energy is carried by the largest scales: $E(k) \propto k^{-5/3}$

Effect of a single turbulence layer

- Within this model, the structure function (related to the covariance) of the refraction index can be predicted: $D_n(\delta x) = C_n^2\, \delta x^{2/3}$
- Refraction index fluctuations result in phase fluctuations, since $d\varphi(x) = 2\pi/\lambda\, n(x)\, dz$
- Near-field: phase fluctuations have the same properties at height *h* and on the ground
  – 2nd order effect: scintillation due to diffraction *if the layer is high enough* (and small telescope)

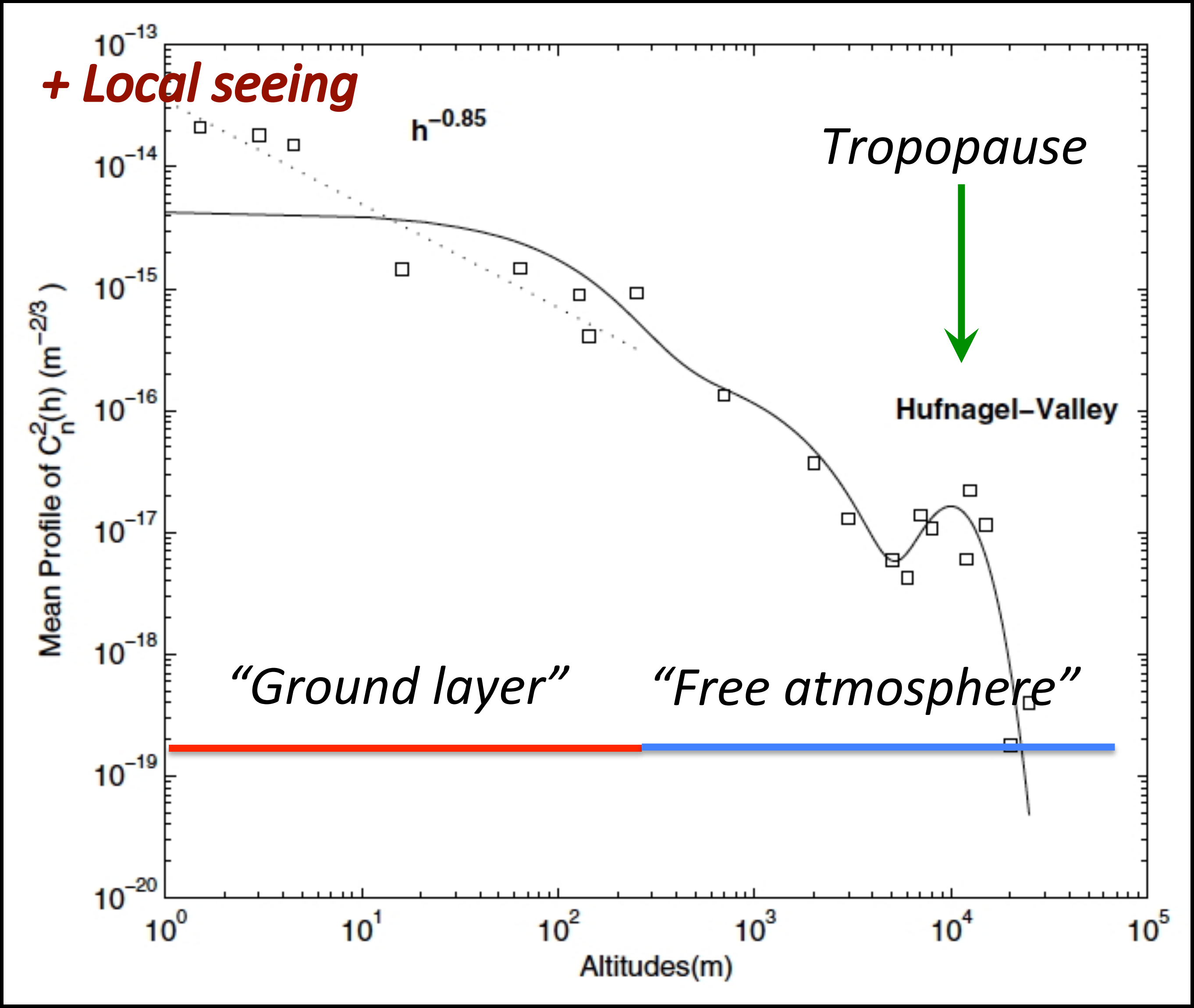
Effect of a single turbulence layer

- For large aperture radii, central limit theorem leads to Gaussian statistics for the phase coherence function (measures the ability of two sub-beams to interfere constructively)
  - $B_\varphi = \exp(-1/2\ D_\varphi)$
- After some algebraic manipulations
  - $D_\varphi(\delta x) = 2.91\ k^2\ C_n^2\ \Delta h\ \delta x^{5/3}$
    - $k = 2\pi / \lambda$
    - Δh = *layer thickness*

Adding all layers

- Simply assume independent horizontal layers, each one adding a separate phase shift at each position of the wavefront
- Previous equation is modified to becomes a continuous integral over z
- All necessary knowledge is encapsulated in the vertical profile of turbulence
  - so-called C_n^2 profile

Breaking down the layers



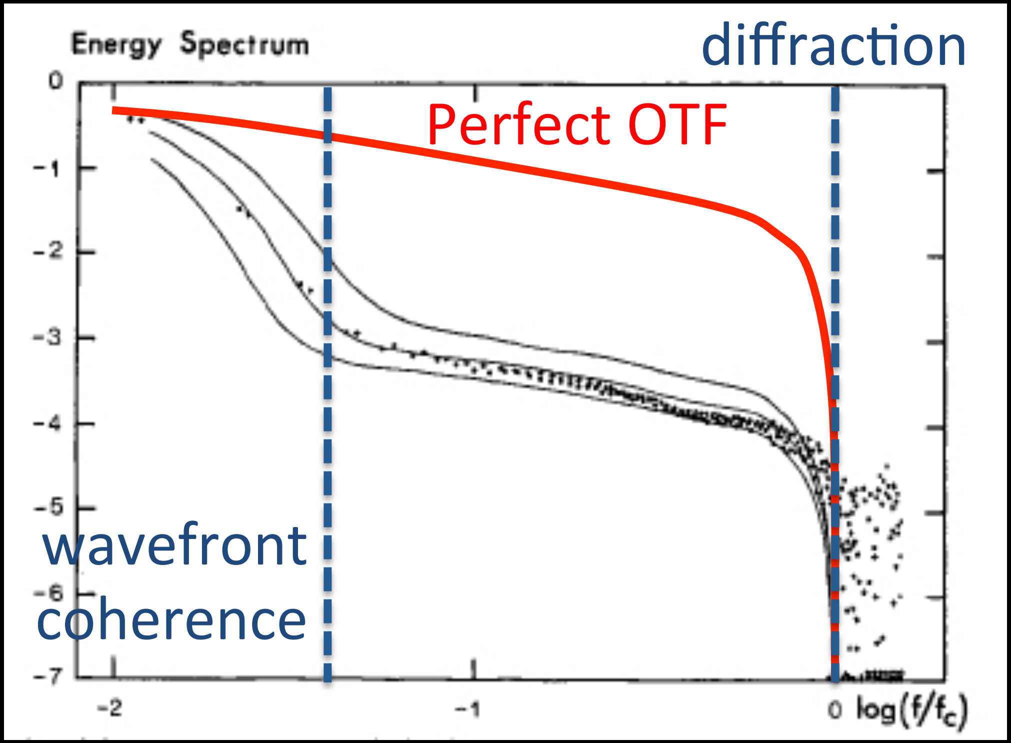
Dali Ali et al. (2010)

Dealing with multiple layers

- Default is to assume that phase screen are conjugated to primary mirror (near-field)
- However, this is not perfect since the atmosphere is not at infinity
- In case a of a dominant layer, better to work in its conjugated plane rather than in the pupil plane

Effect on OTF

- A full spectrum, attenuating the OTF at all frequencies except the shortest ones (where the field remains correlated)



Fried's parameter

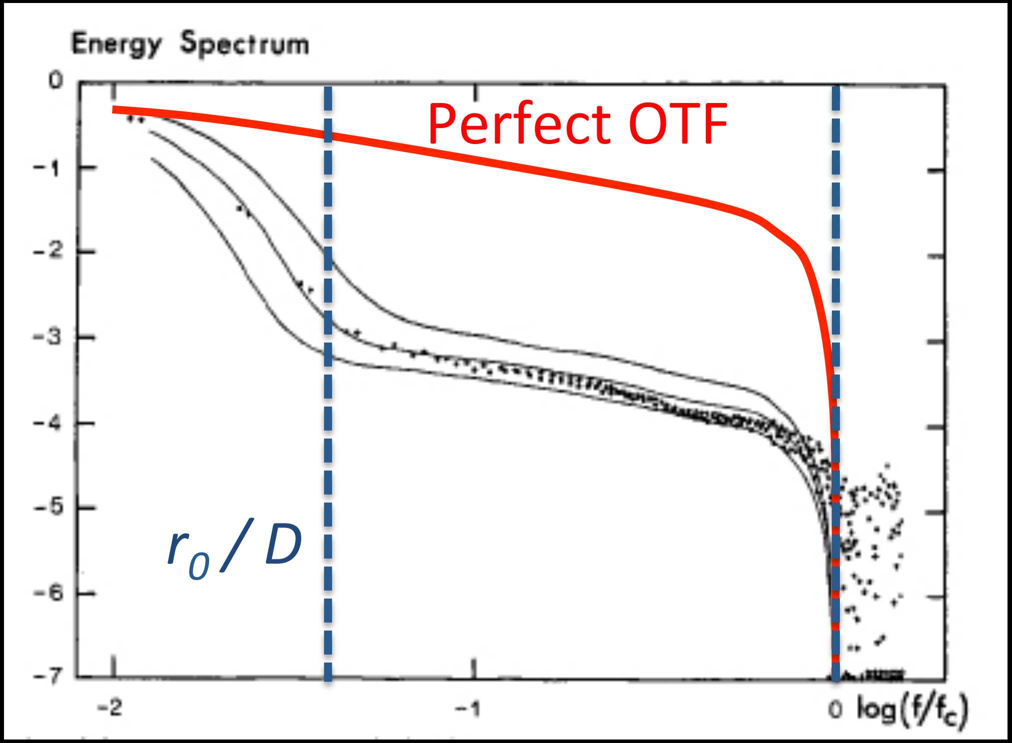
- A natural length scale: *$D\varphi(\delta x) = 6.88\ (\delta x/r_0)^{5/3}$*

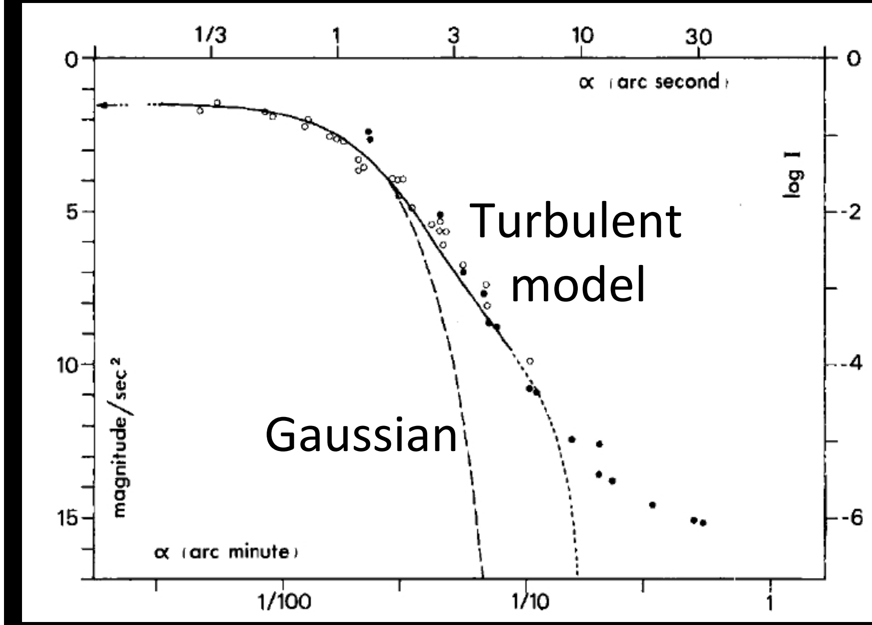
$$r_0 = \left[0.423 k^2 (\cos\gamma)^{-1} \int C_N^2(h)\,\mathrm{d}h\right]^{-3/5}$$

- Length scale over which phase correlation is *$rms(\varphi) \leq 1$*rad (remember Maréchal's criterion!)
- r_0 scales as
  - $\lambda^{6/5}$
  - $(\cos \gamma)^{3/5}$ [γ = zenithal angle (sometimes z)]
  - The integral of C_n^2

Fried's parameter

- r_0 drives the "seeing"
- A point source has a quasi-Gaussian profile





Telescope size and r_0

- Small telescopes: $D < r_0$
  - Yield diffraction limited images
  - Still subject to image jitter (tip-tilt does not decrease wavefront coherence!)
- Large telescopes: $D > r_0$
  - Effect of turbulent decorrelation are important
  - Image shape depends on D/r_0 ratio and on time variability vs exposure time

Short and long exposure image

- Short exposure (“frozen turbulence”) contains information out to the highest frequency
  - It is “diffraction-limited”
  - Independent parts of wavefront form images in different locations: speckle pattern
- Long exposures is sum of random short exposures, with constant phase scrambling
  - All high-frequency info is lost, image is completely “turbulence-limited”

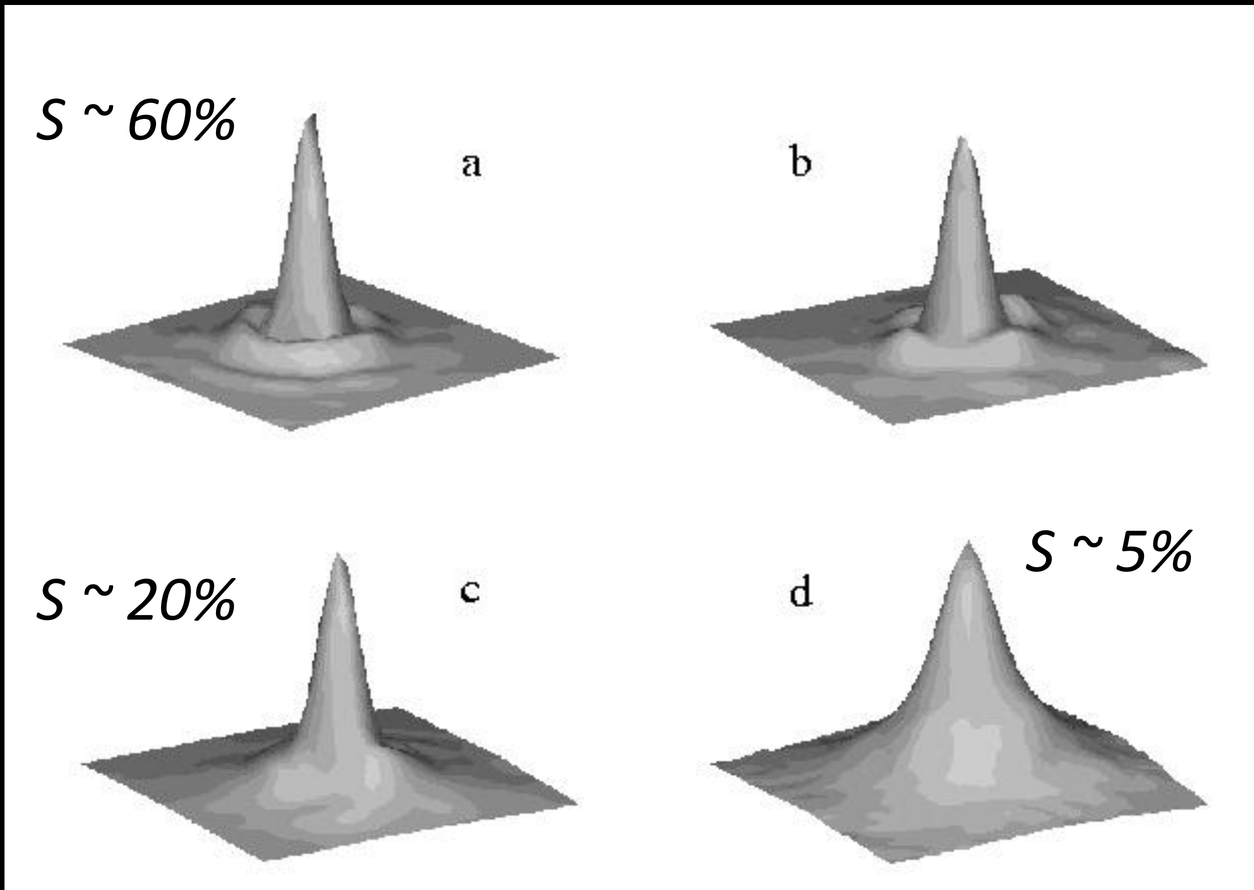
Temporal correlation

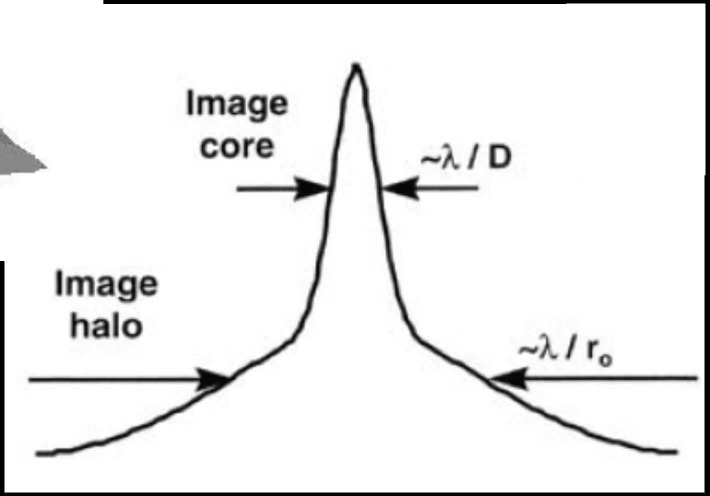
- Frozen flow + single “effective” wind speed (Taylor approximation)
- Timescale over which the phase is decorrelated (*rms* $\varphi = 1$ rad) is the $\tau_0 = 0.314\, r_0 / v$
- Associated frequency is Greenwood frequency
- Any correction needs to be apply at a higher frequency to be successful

Characterizing an aberrated image

- In the image plane, one can measure the PSF:
  - FWHM, detailed image profile, encircled energy …
- A common metric: the Strehl ratio
  - In an image: ratio of peak intensity to that of diffraction-limited image
  - From the OTF: ratio of integral of the OTF with and without aberration
  - Equivalent for diffraction-limited images ($S > 0.1$)

Characterizing an aberrated image





Quantifying atmospheric turbulence

- Fried & Mevers (1974)
- Racine et al. (1991)
- Tokovinin et al. (2003)

Quantifying atmospheric turbulence

- Estimating r_0
  - Measuring C_n^2
  - Fitting the image of a star
  - Measuring (differential) image motion
  - Measuring (differential) scintillation
- Estimating τ_0
  - Assume some effective wind speed
  - Measure temporal coherence of image motion and/or scintillation

Quantifying atmospheric turbulence

- Typical numbers:
  - r_0 = 5-10 cm
  - V = 5-10 m/s
  - τ_0 = 1-5 ms
- Variations on all timescales (min to yr)
- No simple connection to weather
- Relative contributions to seeing vary over time

Next week's readings

- Law et al. (2006)
- Labeyrie (1970)

- Babcock (1953)
- Buffington et al. (1977a,b)
- McCall et al. (1977)

- Hinkley et al. (2007)