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Origin of the metallicity dependence of exoplanet host stars in the protoplanetary disc mass distribution

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ABSTRACT

The probability of a star hosting a planet that is detectable in radial velocity surveys increases as $P_{\rm pl}(Z) \propto (10^Z)^2$, where Z is stellar metallicity. Models of planet formation by core accretion reproduce this trend, since the protoplanetary disc of a high-metallicity star has a high density of solids, and so forms planetary cores which accrete gas before the primordial gas disc dissipates. This paper considers the origin of the form of the metallicity dependence of $P_{\rm pl}(Z)$. We introduce a simple model in which detectable planets form when the mass of solid material in the protoplanetary disc, $M_{\rm s}$, exceeds a critical value. In this model, the form of $P_{\rm pl}(Z)$ is a direct reflection of the distribution of protoplanetary disc masses, $M_{\rm g}$, and the observed metallicity relation is reproduced if $P(M_{\rm g} > M'_{\rm g}) \propto (M'_{\rm g})^{-2}$. We argue that a protoplanetary disc's dust mass measured in submillimetre observations is a relatively pristine indicator of the mass available for planet-building, and find that the disc mass distribution derived from such observations is consistent with the observed $P_{\rm pl}(Z)$ if a solid mass $M_{\rm s} > 0.5M_{\rm J}$ is required to form detectable planets. Any planet formation model which imposes a critical solid mass for detectable planets to form would reproduce the observed metallicity relation, and core accretion models are empirically consistent with such a threshold criterion. While the outcome of planet formation in individual systems is debatable, we identify seven protoplanetary discs which, by rigid application of this criterion, would be expected to form detectable planets and may provide insight into the physical conditions required to form such planets. A testable prediction of the model is that the metallicity dependence should flatten both for $Z > 0.5$ dex and as more distant and lower mass planets are discovered. Further, combining this model with one in which the evolution of a star's debris disc is also influenced by the solid mass in its protoplanetary disc results in the prediction that debris discs detected around stars with planets should be more infrared luminous than those around stars without planets in tentative agreement with recent observations.

Key words: circumstellar matter – planetary systems: formation – planetary systems: protoplanetary discs – stars: pre-main-sequence.

1 INTRODUCTION

The study of how planetary systems form and evolve was revolutionized when the first extrasolar planet was discovered in radial velocity studies of the star 51 Peg (Mayor & Queloz 1995). Over 200 extrasolar planets are now known (Butler et al. 2006), and studying these planets has yielded enormous advances in our understanding of how they formed (Papaloizou & Terquem 2006; Udry, Fischer & Queloz 2007). Perhaps the most telling discovery was that of a correlation in the probability of a star hosting a planet, $P_{\rm pl}$, which is found to increase with stellar metallicity (Gonzalez 1997). Fischer & Valenti (2005, hereafter FV05) found that, for stars with a metallicity Z = [Fe/H] between −0.5 and 0.5 dex, the metallicity dependence of the fraction of stars with planets with orbital periods <4 yr and with amplitudes in radial velocity studies in excess of $K > 30\,{\rm m\,s^{-1}}$ (i.e. Saturn–Jupiter mass planets, depending on orbital period) is

$$P_{\rm pl}(Z) = 0.03 \times 10^{2Z}, \quad (1)$$

which corresponds to a planet fraction which increases with the square of the number of iron atoms in the stellar atmosphere. Similar trends have been found to apply to all species including Si and Ni (e.g. Ecuvillon et al. 2004; Gonzalez 2006; Robinson et al. 2006).

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The origin of this metallicity dependence is thought to be intrinsic to the planet formation process (FV05), and not caused by contamination from planetesimals falling on to the star, as is believed to be the cause of the high metallicities of DAZ white dwarfs (Jura 2006; Kilic 2007), although the recent discovery that planet hosting giant stars do not favour metal rich systems is currently reigniting this debate (Pasquini et al. 2007).

Since the discovery of the extrasolar planet metallicity correlation, much work has gone into considering how stellar metallicity could affect different aspects of the planet formation process in the various models (e.g. Livio & Pringle 2003). It has been found that forming planets by gravitational instability does not introduce any significant metallicity dependence (Boss 2002; Cai et al. 2006), whereas models of planet formation by core accretion seem to readily reproduce the observed trend (Ida & Lin 2004b, hereafter IL04b; Kornet et al. 2005; Benz et al. 2006; Robinson et al. 2006). This is because, in the core accretion models, planetesimals grow into planet cores through collisions, subsequently accreting gas from the surrounding gas disc once they become large enough, and then interacting with that disc so as to migrate inwards (e.g. Lin & Papaloizou 1986; Papaloizou et al. 2007). The core accretion models predict a metallicity dependence because a higher metallicity implies higher solid mass, and hence faster core growth, which means that the critical core mass for gas accretion can occur before the gas disc dissipates on ∼6 Myr time-scales (Clarke, Gendrin & Sotomayer 2001; Haisch, Lada & Lada 2001). However, it remains to be explained why the metallicity dependence has a form $\propto 10^{2Z}$ as opposed to, for example, $\propto 10^{Z}$. The origin of the dependence found in these models is hidden somewhere within the large number of model components of which they are comprised, although it has been shown that a large solid disc mass is required if planets are to form IL04b.

In this paper, we consider the origin of the form of the metallicity dependence using a simple heuristic model in which detectable planets form as long as the solid mass of material in the protoplanetary disc exceeds a critical value (e.g. Greaves et al. 2007). That model is described in Section 2, where it is shown how the metallicity relation is then directly related to the initial disc mass distribution. This section also compares the disc mass distribution required to reproduce the observed planet–metallicity trend in this model with that inferred from submillimetre (submm) observations of star-forming regions. The implications of this model are discussed in Section 3, along with a discussion of why the solid mass should provide such a strong constraint on whether a system goes on to form a detectable planet. The conclusions are given in Section 4.

2 CRITICAL SOLID MASS MODEL

This model assumes that stars form surrounded by a protoplanetary disc which is made up of both solids and gas. We denote the mass of each of these components by $M_{\rm s}$ and $M_{\rm g}$, respectively. The gaseous component dominates the total mass of the disc, and it is assumed that the outcome of the star formation process results in some universal distribution of disc masses (i.e. gas masses), which we define by the probability of any given star having had a protoplanetary disc with a gas mass larger than $M'_{\rm g}$ as $P(M_{\rm g} > M'_{\rm g})$. The solid mass of any given disc is assumed to be directly related to the mass of the gaseous component through the final metallicity of the star (e.g. Greaves et al. 2007):

$$M_{\rm s} = 0.01 M_{\rm g} 10^{Z}. \tag{2}$$

Here, we have assumed that the ratio of gas to solids is 100 for stars formed in a $Z = 0$ environment, consistent with that seen in nearby star-forming regions (James et al. 2006). Thus, it is assumed that stellar metallicities are indicative of the conditions present prior to the formation of the star that continued to be reflected in the composition of the protoplanetary disc, and that exerted no influence over the resulting distribution of protoplanetary disc masses.

The most important assumption is then that all of the stars that have discs with $M_{\rm s}$ larger than some critical value $M_{\rm s,crit}$ go on to form planets which can be detected in radial velocity surveys, that is, $P_{\rm pl} = P(M_{\rm s} > M_{\rm s,crit})$. The physical origin for this critical value is not part of this heuristic model, although it does have a physical motivation based on core accretion models (e.g. IL04b), as discussed in Section 1 and in more detail in Section 3.

2.1 Analytical solution

Since the probability of forming a planet depends only on the solid mass, the critical mass above which the total disc mass (i.e. gas mass) must be to form a planet is dependent on metallicity:

$$M_{\rm g,crit} = 100 M_{\rm s,crit} 10^{-Z}. \tag{3}$$

The gas mass distribution is assumed to be independent of metallicity, and so the probability of any star forming a planet is metallicity dependent, since $P_{\rm pl} = P(M_{\rm g} > M_{\rm g,crit})$. Thus to reproduce equation (1) requires a gas mass distribution in which

$$P(M_{\rm g} > M'_{\rm g}) = 0.03 \left(\frac{100 M_{\rm s,crit}}{M'_{\rm g}}\right)^2, \tag{4}$$

where the critical solid mass required to form a planet, $M_{\rm s,crit}$, is some as yet undefined constant. Since the probability of any star hosting, a planet given in equation (1) is only known to apply for $P_{\rm pl} < 0.25$ (due to the lack of surveys at higher Z), it follows that the distribution given in equation (4) is also only valid for $P(M_{\rm g} > M'_{\rm g}) < 0.25$, and so for $M_{\rm g} > \sqrt{1200} M_{\rm s,crit}$. Thus, in this model the observed $P_{\rm pl}(Z)$ in equation (1) is telling us about the mass distribution of the most massive 25 per cent of discs.

2.2 Gas disc distribution from observations

The gas mass distribution required by this model in order to match the observed $P_{\rm pl}(Z)$ (equation 4) can now be compared with the observed gas mass distribution. The gas mass distribution of protoplanetary discs is not well known because the majority of that mass is in molecular hydrogen which is difficult to detect, especially in the cold outer regions of the discs where most of the mass resides (Thi et al. 2001; Sheret, Ramsay-Howat & Dent 2003). Species such as CO are easier to detect (e.g. Dent, Greaves & Coulson 2005; Dutrey, Guilloteau & Ho 2007); however, there is uncertainty in the CO/H_2 ratio because some of this gas ends up frozen on to dust grains or photodissociated (Dullemond et al. 2007; Najita et al. 2007). On the other hand, the dust mass distribution of protoplanetary discs is well characterized, since this can be measured with relatively few uncertainties from submm and millimetre (mm) wavelength observations (André & Montmerle 1994; Beckwith, Henning & Nakagawa 2000).

Here, we make the assumption that dust mass can be used as a proxy for the total gas mass in protoplanetary discs (for a fixed Z), and we derive the gas mass distribution from the dust mass distribution in Taurus–Auriga, which was measured using submm photometry of 153 pre-main-sequence stars by Andrews & Williams (2005,

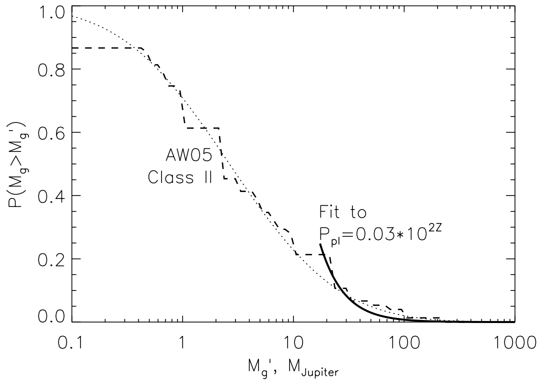

Figure 1. Distribution of protoplanetary disc gas masses. The gas mass distribution inferred from the dust mass distribution of class II objects in Taurus–Auriga (Andrews & Williams 2005) is shown with a dashed line. The dotted line is a lognormal fit to this distribution centred on $2.5M_J$ of width 0.77 dex. The distribution required in the critical solid mass model to fit the extrasolar planet metallicity relation (equations 1 and 4) is shown with a solid line, assuming $M_{s,crit} = 0.5M_J$.

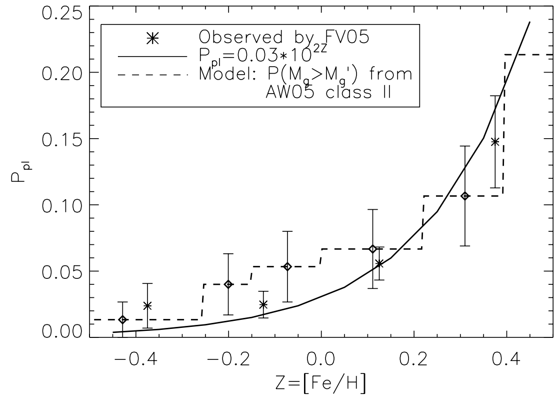

Figure 2. Probability that a star of given metallicity has an extrasolar planet that is detectable in the current radial velocity surveys. The predictions of the critical solid mass model based on the Andrews & Williams (2005) distribution of dust masses of class II protoplanetary discs in Taurus–Auriga are shown with a dashed line, with errors indicated by diamonds with $\sqrt{N}$ error bars. The asterisks show the results of the radial velocity survey of Fischer & Valenti (2005) with $\sqrt{N}$ error bars. The fit to the FV05 data (equation 1) is shown with a solid line.

hereafter AW05). Since the stars in the AW05 sample are at a range of evolutionary stages, we chose to use only the disc masses of the 75 class II objects (i.e. T Tauri stars) in their sample to ensure that the disc mass distribution is indicative of that at the epoch of planet formation. Class I sources were omitted because of a potential contribution to the submm flux from a remnant circumstellar envelope. Class III sources were omitted because of the possibility that their currently low disc masses are a consequence of the discs being at an advanced evolutionary stage, and so are not necessarily indicative of a low mass present at the planet-forming epoch. To obtain the gas mass distribution, the gas-to-dust ratio was assumed to be 100 for all stars, based on the metallicities in nearby star-forming regions being close to solar with a small dispersion for each region (Padgett 1996; Vuong et al. 2003; James et al. 2006). The mass distribution of class II objects is shown in Fig. 1. Ten objects from this sample have only upper limits to their disc masses, which were set to zero in Fig. 1. Since these upper limits are $\leqslant 1M_J$, we infer that the disc mass distribution is accurate for the most massive 69 per cent (52/75) of discs that are above this limit ($\geqslant 1M_J$).[1]

The critical solid mass model (Section 2.1) was used to determine the metallicity relation predicted from the observed gas mass distribution:

$$P_{pl} = \frac{N(M_g > M'_g) \pm \sqrt{N(M_g > M'_g)}}{N_{tot}}, \quad (5)$$

where Poisson counting statistics were used to determine the uncertainty in the number of discs larger than a given limit in the distribution and $N_{tot} = 75$. The probability determined from equation (5) could be assigned a corresponding metallicity, Z', from the relation $M'_g = M_{g,crit}$. Equation (3) means that

$$Z' = \frac{-\log 0.01M'_g}{M_{s,crit}}. \quad (6)$$

[1] We note that, even though 10 of the AW05 class II sources were not detected in individual submm photometry observations, co-addition of this data set leads to a net positive detection of 2.7 ± 0.9 mJy, corresponding to a mean disc mass of $0.14M_J$ which is consistent with that expected from the lognormal distribution plotted in Fig. 1.

The value of $M_{s,crit}$ was constrained to achieve a mean planet probability for the metallicity range $Z = 0.25$–0.5 dex in agreement with that found by FV05, that is, $P_{pl} = 14.8 \pm 3.5$ per cent, giving[2]

$$M_{s,crit} = (0.5 \pm 0.1)M_J. \quad (7)$$

The extrasolar planet–metallicity relation predicted by this model is plotted in Fig. 2, and shows good agreement with the observed relation (equation 1).

We have also inverted the problem by deducing the *required* disc mass distribution that would lead to the solid line in Fig. 2 [i.e. $P_{pl}(Z)$ parametrized according to equation 1]. In Fig. 1, we compare this required distribution with the observed gas mass distribution. Noting that this comparison can only be made over the upper quartile of disc masses (since current planet detection statistics only extend to metallicities < 0.5 and, in the model, it is only this range of disc masses which can form planets in this metallicity regime), it is evident that there is also good agreement between the model and the observed distributions when plotted in this way. To quantify this, we performed a one sided Kolmogorov–Smirnov test to compare the distribution of gas masses inferred from AW05, when converted into metallicity (equation 6), with that inferred from equation (1) for the range $Z' = -0.5$ to 0.5 dex. We found that discrepancies as large as or greater than those observed occur in 69 per cent of samples of 75 members drawn from a population with a cumulative distribution function in which $P(Z < Z') = 0.03 \times 10^{2Z'}$; that is, we conclude that the gas mass data are not unlikely to be drawn from such a distribution, since at least two out of three times one would expect data at least as discrepant as observed.

3 DISCUSSION

We have shown, under the assumption that a critical solid mass in the protoplanetary disc is required to form a planet that is detectable in radial velocity surveys, that the observed frequency of planet detections as a function of metallicity, $P_{pl}(Z)$, is compatible with

[2] The value derived in equation (7) differs slightly from $0.24M_J$ derived by Greaves et al. (2007) because that paper included discs from AW05 of both classes II and III in their primordial gas mass distribution.

the observed disc mass distribution (as derived from submm dust mass measurements of classical T Tauri stars in local star-forming regions). We now discuss the physical basis for this simple model and further observational tests.

3.1 Comparison with core accretion models

To consider the physical basis for the outcome of planet formation being determined solely by dust mass, we appeal to the core accretion models of (Ida & Lin 2004a, hereafter IL04) and IL04b. The IL04 models are *local*, in the sense that planet formation depends on local quantities such as gas and solid surface densities. Therefore, we expect any threshold effect to involve surface density rather than mass. We first assess whether the results of IL04 are compatible with the hypothesis that planet formation requires a critical *metallicity independent* solid surface density and return to a discussion of the relationship between solid surface density normalization and dust mass in Section 3.2. We can assess this hypothesis in two ways. First, we can simply take the distribution of disc surface densities assumed by IL04 (a lognormal distribution of width 1.0 dex that is centred on the surface density of the minimum mass solar nebula and truncated at $>1.48\sigma$), apply a threshold solid surface density for planet formation that is independent of metallicity and see whether we can reproduce their numerical results. Fig. 3 shows that this is indeed the case: the nominal model from IL04b is well reproduced by assuming a critical solid surface density of eight times the minimum mass solar nebula, whereas their variant models where the rate of core accretion is enhanced or reduced by a factor of 3 are well reproduced by models in which the critical solid surface density is, respectively, four and 22 times the minimum mass solar nebula. We stress that the IL04 models contain a large number of ingredients and do not explicitly impose a threshold criterion. Nevertheless, we see that their results are empirically equivalent to the imposition of a simple threshold.

In a second approach, we can now attempt to understand *why* the IL04 models behave in this way. Examination of these models shows that the formation of gas giant planets hinges on rocky cores being able to grow to a critical mass (a few $M_{\odot}$) before the gas disc is dispersed. The requirement of sufficiently rapid core growth implies that they have to form inside a critical radius, $a_{\rm ig}$, which depends on both gas and solid surface densities. On the other hand, inward of a second critical radius, $a_{\rm tg}$ (which depends on solid surface density), a critical core mass is not achievable because the required core mass exceeds the local isolation mass (at which point the core has consumed all the material in its local feeding zone). Evidently, the formation of gas giant planets is possible only for the case $a_{\rm tg} < a_{\rm ig}$ and we can derive a condition on the gas and the solid surface densities corresponding to the critical case, where $a_{\rm tg} = a_{\rm ig}$. This translates into a condition on the minimum surface density of solids as a function of metallicity. We find that the critical surface density of solids scales as $10^{-0.06Z}$ (assuming, as in IL04, that a disc's surface density scales $\Sigma \propto r^{-p}$, where r is radius and $p = 1.5$). This very weak dependence on metallicity results from the fact that the growth rate of solid cores is much more strongly dependent on orbital radius than on the gas column density, and hence $a_{\rm ig}$ is only very weakly dependent on gas column density. Therefore, the threshold criterion $a_{\rm ig} = a_{\rm tg}$ is nearly independent of gas column density, and thus the dependence of critical dust column on metallicity is extremely weak. It is this extremely weak dependence of the critical solid surface density on metallicity which we believe to account for the excellent correspondence between the numerical results of IL04 and the application of our simple threshold hypothesis (see Fig. 3).

A further test of this hypothesis would be to examine how $P_{\rm pl}(Z)$ predicted by the core accretion models depends on the assumed distribution of disc surface densities, since if the outcome is governed by a critical surface density of solids for planet formation then using a narrower distribution of disc surface densities as input would result in a steeper metallicity dependence (since in the critical solid surface density model the metallicity dependence simply reflects the disc surface density distribution used as input). In contrast to IL04, Robinson et al. (2006) did vary this quantity and indeed found that $P_{\rm pl}(Z)$ rose more gently when a larger range of disc surface densities was employed.

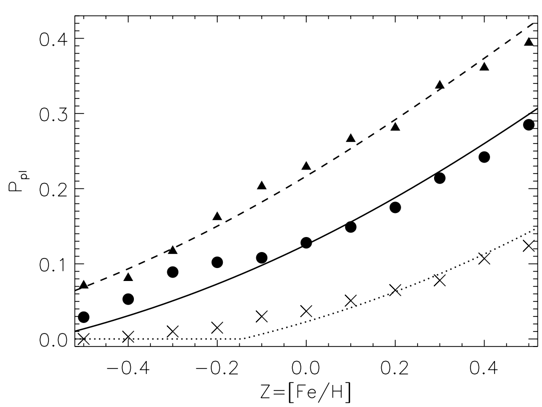

Figure 3. Prediction of the critical solid mass model for the probability that a star of given metallicity has an extrasolar planet that is detectable in the current radial velocity surveys assuming the disc mass distribution used in IL04b with critical solid masses of four (dashed line), eight (solid line) and 22 (dotted line) times the minimum mass solar nebula. The numerical results of the nominal model in IL04b are shown with filled circles, and the results of their models in which the core accretion rate is three times faster and slower than the nominal model shown with triangles and crosses (see their fig. 2b).

3.2 Why submm dust mass determines outcome

Regardless of the comparison with core accretion models, it is notable that the critical solid mass model fits the planet–metallicity relation found in nature. It is, however, surprising that submm dust mass should be such a good indicator of whether planets are going to form in a disc, since submm measurements probe the current mass in mm- to cm-sized dust and so are not necessarily representative of the primordial inventory of solid or gas mass. Indeed, class II objects in Taurus–Auriga have a range of ages and so we would expect the oldest stars to have already lost a significant quantity of gas through accretion on to the star (Clarke et al. 2001). We may also expect some loss of detectable dust mass with age through grain growth and accretion on to the star with the gas. However, there is no evidence that submm dust mass changes with age on the pre-main sequence (e.g. Wyatt, Dent & Greaves 2003) suggesting that the mass in mm to cm sizes is constant. This is to be expected, since the total dust mass $M_{\rm dust} \propto r_{\rm out}^{2-p}$, where $r_{\rm out}$ is the disc outer edge, so that as long as $p < 2$ the submm dust mass is concentrated in the outer regions of the disc. Since typically observed values for protoplanetary discs are $p \approx 0.85$ and $r_{\rm out} \approx 200$ au (Andrews & Williams 2007), the time-scale for grains containing most of the disc mass to grow to larger than 1 m, and so become invisible in the submm, may be expected to be longer than the 10 Myr period over which planet formation (in the inner regions) must take place (e.g. Dullemond & Dominik 2005). Indeed some discs cannot harbour significant quantities of 'unseen' dust mass (i.e. with particle sizes either much larger or smaller than 1 mm), since, even in the absence of such unseen contributions, the gas mass inferred from mm dust measurements is in some cases already ~0.2 times the central star's mass, and thus close to the limit for gravitational instability. Given the evidence that grain growth to mm and cm scales has occurred in the outer regions of discs (Wilner et al. 2005), we are confident that this grain size scale contains the majority of the disc solid mass at these radii, and thus, by implication, the majority of the solid mass in the disc. Thus, while the dust seen in the submm is not contributing to the planet formation process (because it is mainly at radii where it has not had time to grow to large – greater than metre – size scales), we are suggesting that it is nevertheless a good measure of the primordial inventory of solids in the disc.

The fact that the submm dust mass distribution fits the observed planet–metallicity relation so well is because there is an order of magnitude difference between the highest and the lowest masses of the top ~25 per cent most massive gas discs (e.g. Figs 1 and 2). This result is not specific to the Taurus–Auriga star-forming region, since class II discs in ρ Oph also exhibit an order of magnitude range for the most massive 25 per cent of those discs (see fig. 9 of André & Montmerle 1994). If this distribution had been much narrower or broader then we would have been able to rule out the critical solid mass model.

One further requirement of nature for the critical solid mass model to work is for a disc's outer radius to be less important than its solid mass in setting the outcome of planet formation. As noted in Section 3.1, models such as those in IL04 rely on a critical surface density (rather than mass). For the surface density profile assumed by IL04, the surface density normalization (f_{d}, where $\Sigma \propto f_{\mathrm{d}}$), disc outer radius ($r_{\mathrm{out}}$) and total solid mass ($M_{\mathrm{s}}$) are related via $M_{\mathrm{s}} \propto f_{\mathrm{d}} r_{\mathrm{out}}^{0.5}$. Thus, the mapping between critical surface density and critical mass is (weakly) dependent on r_{out}. While disc radii have been measured using submm interferometry (Kitamura et al. 2002; Andrews & Williams 2007), these samples are biased towards the most massive discs so that it is not clear how representative the observed distribution is of the population as a whole. However, there is no evidence that the distribution of r_{out} is as broad as that of disc masses seen by AW05. We therefore expect the surface density of solids in the planet formation region to be mainly controlled by M_{s} rather than r_{out}, thus explaining the apparent success of submm flux as a predictor of planet-forming potential.

3.3 Discs forming detectable planets

One implication of this study is that we can predict which of the discs in the AW05 sample will go on to form planets like those detected in the current radial velocity surveys. The class IIs in their sample with more than $0.5M_{\mathrm{J}}$ of dust are 04113+2758, DL Tau, GG Tau and GO Tau. However, we disqualify GG Tau as a planet-forming candidate, since its disc is circumbinary (Guilloteau, Dutrey & Simon 1999), and so its high submm flux does not equate with a high surface density of solids in the inner disc. Massive circumbinary discs are rare (Jensen, Mathieu & Fuller 1996), so the majority of the more massive discs are not circumbinary discs and so would not be unsuitable for forming planets. Applying the same $0.5M_{\mathrm{J}}$ dust mass limit to the ρ Oph study of André & Montmerle (1994) indicates that of the class IIs in this region, AS205, EL24, GSS39 and SR24S may go on to form detectable planets.

While we do not claim that we can unambiguously predict the outcome of planet formation for any one of these systems, we do suggest that studying the discs that are predicted to form planets, the characteristics of which we can constrain at least statistically, may provide a valuable way of probing the environments in which such planets form. The fact that it is the most massive discs which go on to form detectable planets means that these discs must be close to being gravitationally unstable, since the ratio $M_{\mathrm{disc}}/M_{\star} > 0.05$ for $Z = 0$ and $M_{\star} = 1\,\mathrm{M}_{\odot}$. This suggests that instability could play a role in the formation process. However, this cannot be the only determining factor, since the gravitational instability process itself is not affected by metallicity (Cai et al. 2006), and there would be no metallicity dependence if $M_{\mathrm{g,crit}}$ is a constant and not dependent on metallicity. Thus, this suggests that some degree of instability may help speed up the core accretion process, for example, through concentration of particles in spiral structures (Rice et al. 2004) or instability in a thin dust layer (Youdin & Shu 2002).

3.4 Observational tests

Here, we suggest three observational tests of the critical solid mass model.

First, if the model is correct, we would expect $P_{\mathrm{pl}}(Z)$ to rise much less steeply with Z at metallicities above 0.5 dex than implied by an extrapolation of equation (1), since at higher metallicities the model predicts that planets would be able to form in lower mass discs, and that $P_{\mathrm{pl}}(Z)$ in this regime would reflect the disc mass distribution of intermediate mass discs. A discrepancy between the observed disc mass distribution and that resulting from an extrapolation of equation (1) to $Z > 0.5$ dex is readily apparent by considering how the solid curve on Fig. 1, if extrapolated to lower disc masses, would compare with the dashed line on that figure. Whether a suitable high-metallicity sample can be found to test this prediction remains to be seen (e.g. Laughlin 2000; Valenti & Fischer 2005; Taylor 2006).

Secondly, one of the key assumptions of the model was that the distribution of protoplanetary disc masses is universal in that it is independent of metallicity. This can be tested by measuring the distribution of dust masses in low- (or high-) metallicity star-forming regions using submm photometry, since these masses should be correspondingly lower (or higher) than those of nearby regions like Taurus–Auriga where $Z \approx 0$. While the Atacama Large Millimetre Array (ALMA) can detect the brightest known class II discs out to 20 kpc, we are not aware of any young (<10 Myr) cluster within the Milky Way which has a measured metallicity that is sufficiently sub- or super-solar for the predicted difference in disc mass distribution in comparison with Taurus–Auriga to be confidently detected, although star-forming clusters such as those found by Santos et al. (2000) and Yun, Lopez-Sepulcre & Torrelles (2007) may be suitable candidates if their large Galactocentric distances (15–16.5 kpc) are indicative of a low metallicity as suggested by observations Cepheids which indicate a metallicity gradient in the Milky Way of $-0.06\ \mathrm{dex\,kpc^{-1}}$ (Luck, Kovtyukh & Andrievsky 2006).

Thirdly, we will be able to test in due course an adjunct hypothesis, that is, that the incidence of planets of lower masses (and at greater orbital distances) is also regulated by a (lower) critical solid mass threshold. For example, extrapolation of the exoplanet semi-major axis distribution to 20 au suggests that surveys able to detect planets to that distance would double the fraction of stars known to have planets to 12 per cent (Marcy et al. 2005). The simplest hypothesis we can apply to this population would simply be that the progenitor discs corresponded to the top 12 per cent of the disc mass distribution, implying a critical solid mass of ~$0.3M_{\mathrm{J}}$. We plot in Fig. 4 the predicted dependence of planet frequency on metallicity in this case. Although this 'prediction' will eventually be compared with observational data, we emphasize that it is not entirely clear

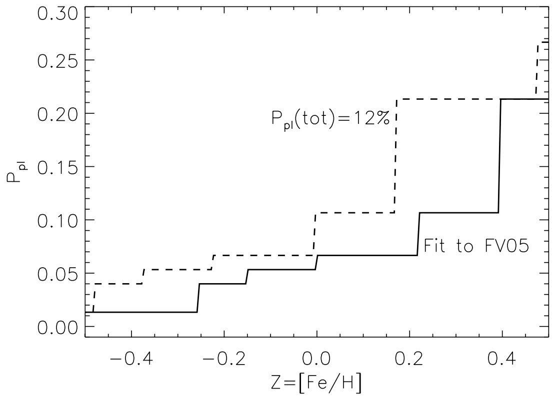

Figure 4. Prediction of the critical solid mass model for the probability that a star of given metallicity has an extrasolar planet. Assuming that surveys with different detection thresholds correspond to different critical solid masses, the prediction for planets that are detectable in the current radial velocity surveys is shown with a solid line, and the prediction for a survey with an overall planet detection frequency of 12 per cent for the metallicity range −0.5 to 0.5 dex is shown with a dashed line.

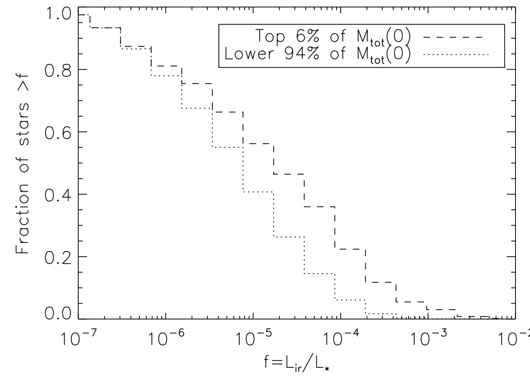

Figure 5. Distribution of infrared luminosities ($f = L_{ir}/L_\star$) of the debris discs of A stars in the model of Wyatt et al. (2007). The distribution for the debris discs formed from the most massive 6 per cent of protoplanetary discs (the *planet bearers*) is compared with those formed from the least massive 94 per cent of protoplanetary discs (the *non-planet bearers*).

how this adjunct hypothesis (i.e. that the critical solid mass is lower for planets located at larger distances) can be squared with the expectations of core accretion models.

4 CONCLUSIONS

We have presented a simple analytical model which can be used to predict the outcome of planet formation, in which the formation of a planet that is detectable in radial velocity studies depends only on the mass of solids in the protoplanetary disc. We showed that this model predicts that the observed planet–metallicity relation is a reflection of the disc mass distribution. We also argued that the submm dust mass seen in the protoplanetary disc phase is a good tracer of the initial mass budget available close to the star for planet formation, and showed that the observed planet–metallicity relation is consistent with the disc mass distribution estimated from submm observations of protoplanetary discs if the critical solid mass required to form detectable planets is $0.5M_J$.

We suggested that the detailed physics of the IL04 core accretion models boil down to a critical solid mass required to form detectable planets, although it needs to be confirmed that the good empirical agreement with the IL04 models is more than a coincidence. However, the value of this model is not just in its relevance to specific core accretion models, but in its general applicability, since it shows how the observed planet–metallicity relation would be reproduced by any planet formation model which imposes a critical solid mass for the formation of detectable planets. Other reasons for imposing a threshold on a disc's solid mass before detectable planets can form include the possibility that such conditions are required for the formation of greater than km-sized planetesimals through gravitational instabilities (e.g. Johansen, Klahr & Henning 2006).

The value of this model is also in its simplicity, since this means that it can be readily applied to predict other observable properties of stars with and without detectable planets, should those properties also depend on the solid mass of the protoplanetary disc. For example, the statistics for the incidence of debris discs around A stars as a function of stellar age (e.g. Rieke et al. 2005) can be explained by a model in which all stars form planetesimal belts, the initial mass of which is determined by the solid mass in the protoplanetary disc (a distribution which is taken from AW05), and which are subsequently eroded by steady state collisional processing (Wyatt et al. 2007). We ran the A star debris disc model in order to predict the distribution of $f = L_{ir}/L_\star$ for the ensemble, computing separate distributions for the *planet bearers* (corresponding to the top 6 per cent of the input mass distribution) and the *non-planet bearers* (corresponding to the remaining 94 per cent of the population). Fig. 5 shows how the debris discs of the *planet bearers* are, on average, more luminous than those of the *non-planet bearers*; specifically, the mean luminosity of the most luminous 10 per cent in both distributions (in $L_{ir}/L_\star$) differ by a factor of ∼6. While we do not know whether it is only the top 6 per cent of A star protoplanetary discs that form detectable planets, because the A star exoplanet population is poorly known at present, here we predict that if A star planets form in a similar manner to those of sun-like stars (i.e. with a threshold solid mass criterion), then this will be seen in the luminosity distributions of their debris discs (Fig. 5). Similarly, while we do not know whether the luminosity distributions of the debris discs of sun-like stars behave as shown in Fig. 5, because the model of Wyatt et al. (2007) has yet to be applied to that population, here we predict that if the luminosities of sun-like star debris discs are governed by steady state processes, then the distributions of those luminosities will exhibit a trend similar to that in Fig. 5. Indeed observations of the debris discs of sun-like stars both with and without planets do show a trend in their luminosities of comparable magnitude to that suggested by Fig. 5 (Bryden et al., in preparation).

Application of this model to known systems implies that the discs of 04113 + 2758, DL Tau, GO Tau, AS205, EL24, GSS39 and SR24S will form (or have formed) gas giant planets. While the outcome of planet formation in individual systems is uncertain, we suggest that studying these discs may help constrain the physical conditions of discs in which we know, at least statistically, what the outcome of planet formation will be. Observational tests of the model include a flattening of the metallicity relation for $Z > 0.5$ dex and also a flattening as planet search continues.

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REFERENCES

André P., Montmerle T., 1994, ApJ, 420, 837
Andrews S. M., Williams J. P., 2005, ApJ, 631, 1134 (AW05)

Andrews S., Williams J., 2007, ApJ, 659, 705
Beckwith S. V. W., Henning T., Nakagawa Y., 2000, in Mannings V., Boss A. P., Russell S. S., eds, Protostars and Planets IV. Univ. of Arizona Press, Tucson, p. 533
Beichman C. A. et al., 2006, ApJ, 652, 1674
Benz W., Mordasini C., Alibert Y., Naef D., 2006, in Arnold L., Bouchy F., Moutou C., eds, Tenth Anniversary of 51 Peg-b: Status of and Prospects for Hot Jupiter Studies. Frontier Group, Paris, p. 24
Boss A., 2002, ApJ, 567, L149
Butler R. P. et al., 2006, ApJ, 646, 505
Cai K., Durisen R. H., Scott M., Boley A. C., Mejía A. C., Pickett M. K., D'Alessio P., 2006, ApJ, 636, L149
Clarke C. J., Gendrin A., Sotomayer M., 2001, MNRAS, 328, 485
Dent W. R. F., Greaves J. S., Coulson I. M., 2005, MNRAS, 359, 663
Dullemond C. P., Dominik C., 2005, A&A, 434, 971
Dullemond C. P., Hollenbach D., Kamp I., D'Alessio P., 2007, in Reipurth B., Jewitt D., Keil K., eds, Protostars and Planets V. Univ. of Arizona Press, Tucson, p. 555
Dutrey A., Guilloteau S., Ho P., 2007, in Reipurth B., Jewitt D., Keil K., eds, Protostars and Planets V. Univ. of Arizona Press, Tucson, p. 495
Ecuvillon A., Israelian G., Santos N. C., Mayor M., Villar V., Bihain G., 2004, A&A, 426, 619
Fischer D. A., Valenti J., 2005, ApJ, 622, 1102 (FV05)
Gonzalez G., 1997, MNRAS, 285, 403
Gonzalez G., 2006, PASP, 118, 1494
Greaves J. S., Fischer D., Wyatt M. C., Beichman C. A., Bryden G., 2007, MNRAS, 378, L1
Guilloteau S., Dutrey A., Simon M., 1999, A&A, 348, 570
Haisch K. E., Lada E. A., Lada C. J., 2001, ApJ, 553, L153
Ida S., Lin D. N. C., 2004a, ApJ, 604, 388 (IL04)
Ida S., Lin D. N. C., 2004b, ApJ, 616, 567 (IL04b)
James D. J., Melo C., Santos N. C., Bouvier J., 2006, A&A, 446, 971
Jensen E. L. N., Mathieu R. D., Fuller G. A., 1996, ApJ, 458, 312
Johansen A., Klahr H., Henning T., 2006, ApJ, 636, 1121
Jura M., 2006, ApJ, 653, 613
Kilic R. S., 2007, ApJ, 660, 641
Kitamura Y., Momose M., Yokogawa S., Kawabe R., Tamura M., Ida S., 2002, ApJ, 581, 357
Kornet K., Bodenheimer P., Rózczka M., Stepinski T. F., 2005, A&A, 430, 1133
Laughlin G., 2000, ApJ, 545, 1064
Lin D. N. C., Papaloizou J., 1986, ApJ, 309, 846
Livio M., Pringle J. E., 2003, MNRAS, 346, L42
Luck R. E., Kovtyukh V. V., Andrievsky S. H., 2006, AJ, 132, 902
Marcy G., Butler R. P., Fischer D., Vogt S., Wright J. T., Tinney C. G., Jones H. R. A., 2005, Prog. Theor. Phys. Suppl., 158, 24
Mayor M., Queloz D., 1995, Nat, 378, 355
Najita J. R., Carr J. S., Glassgold A. E., Valenti J. A., 2007, in Reipurth B., Jewitt D., Keil K., eds, Protostars and Planets V. Univ. of Arizona Press, Tucson, p. 507
Padgett D. L., 1996, ApJ, 471, 847
Papaloizou J. C. B., Terquem C., 2006, Rep. Prog. Phys., 69, 119
Papaloizou J. C. B., Nelson R. P., Kley W., Masset F. S., Artymowicz P., 2007, in Reipurth B., Jewitt D., Keil K., eds, Protostars and Planets V. Univ. of Arizona Press, Tucson, p. 655
Pasquini L., Döllinger M. P., Weiss A., Girardi L., Chavero C., Hatzes A. P., da Silva L., Setiawan J., 2007, A&A, in press (astro-ph/0707.0788)
Rice W. K. M., Lodato G., Pringle J. E., Armitage P. J., Bonnell I. A., 2004, MNRAS, 355, 543
Rieke G. H. et al., 2005, ApJ, 620, 1010
Robinson S. E., Laughlin G., Bodenheimer P., Fischer D., 2006, ApJ, 643, 484
Santos C. A., Yun J. L., Clemens D. P., Agostinho R. J., 2000, ApJ, 540, L87
Sheret I., Ramsay-Howat S. K., Dent W. R. F., 2003, MNRAS, 343, L65
Taylor B. J., 2006, MNRAS, 368, 1880
Thi W. F. et al., 2001, ApJ, 561, 1074
Udry S., Fischer D., Queloz D., 2007, in Reipurth B., Jewitt D., Keil K., eds, Protostars and Planets V. Univ. of Arizona Press, Tucson, p. 685
Valenti J. A., Fischer D. A., 2005, ApJS, 159, 131
Vuong M. H., Montmerle T., Grosso N., Fiegelson E. D., Verstraete L., Ozawa H., 2003, A&A, 408, 581
Wilner D. J., D'Alessio P., Calvet N., Claussen M. J., Hartmann L., 2005, ApJ, 626, L109
Wyatt M. C., Dent W. R. F., Greaves J. S., 2003, MNRAS, 342, 876
Wyatt M. C., Smith R., Su K. Y. L., Rieke G. H., Greaves J. S., Beichman C. A., Bryden G., 2007, ApJ, 663, 365
Youdin A. N., Shu F. H., 2002, ApJ, 580, 494
Yun J. L., López-Sepulcre A., Torrelles J. M., 2007, A&A, 471, 573

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