Problem Set 3 Solution

AY 7B

Problem 1

In this problem, we are using the equation:

$$\frac{E_{J-K}}{E_{B-V}} = \frac{A_J}{E_{B-V}} - \frac{A_K}{E_{B-V}}, \tag{1}$$

The values of $\frac{A_J}{E_{B-V}}$ and $\frac{A_K}{E_{B-V}}$ can be read of the graph given in class.

We can therefore solve:

$$E_{B-V} = \frac{E_{J-K}}{\frac{A_J}{E_{B-V}} - \frac{A_K}{E_{B-V}}}. \tag{2}$$

Now,

$$R = \frac{A_V}{E_{B-V}} = 3.1, \tag{3}$$

So:

$$A_V = 3.1E_{B-V} = 3.1 \times \frac{E_{J-K}}{\frac{A_J}{E_{B-V}} - \frac{A_K}{E_{B-V}}}. \tag{4}$$

The numerical value would depend on the accuracy of reading off $\frac{A_J}{E_{B-V}}$ and $\frac{A_K}{E_{B-V}}$ from the graph, although it should be around 3.72.

Problem 2: C&O 3.9

(a) Using the usual luminosity equation:

$$L = 4\pi R^2 \sigma T_e^4 = 30500 L_\odot. \tag{5}$$

(b) $M = M_{sun} - 2.5 log(L/L_\odot) = -6.47$, where we have used that $M_{sun} = 4.74$.

(c) $m = M + 5 log(d/10pc) = -1.02$.

(d) $m - M = 5.45$.

(e) $F = \sigma T_e^4 = 3.48 \times 10^{10}\ W\ m^{-2}$.

(f) $F = \frac{L}{4\pi r^2} = 6.5 \times 10^{-8}\ W\ m^{-2}$, which is 4.7×10^{-11} the solar constant.

Problem 3: C&O 3.18

Using Eq. (3.33) and Example 3.6.2,

$$U - B \;=\; -2.5 \log \left(\frac{B_{365} \Delta\lambda_U}{B_{440} \Delta\lambda_B} \right) + C_{U-B}, \tag{6}$$

and similarly for $B - V$. Here, B_λ is the Planck function evaluated at the central wavelength λ, which are 365 nm for U, 440 nm for B, and 550 nm for V. The corresponding filter widths, $\Delta\lambda$'s, are 68 nm, 98 nm, and 89 nm for U, B, and V, respectively. Using Eq. (3.22) for the Planck function at $T = 5777\ K$, we obtain:

$U - B = -0.22$, significantly different from the sun's $+0.195$, and:

$B - V = +0.57$, quite simillar to the actual $+0.65$

Problem 4: C&O 3.19

(a) Following the procedure for Problem 3.18 with $T = 22000\ K$ gives extimates of $U - B = -1.08$ (actual value is -0.90) and $B - V = -0.23$ (the value measured for Shaula).

(b) A parallax angle of 0.00464" implies a distance to Shaula of $d = 1/p = 216pc$. From Eq. (3.6) and $V = 1.62$, $M_V = -5.05$.

Problem 5: C&O 12.1

(a) According to Eq. (12.1), $m_V = M_V + 5 \log d - 5 + a_V = -1.1 + 5 \log 700 - 5 = 8.1$ mag. ($a_V = 0$.)

(b) Using $a_V = 1.1$ mag, $m_V = 9.2$ mag.

(c) If extinction is not considered, using $m_V = 9.2$ mag leads to a distance estimate of 1160 pc (using Eq. 3.5), which is in error by about 65.7%

Problem 6: C&O 12.3

The hydrogen atom's ground-state spin flip produces a photon of frequency $\nu = 1420\ MHz$. This implies that the energy difference between the two energy levels is $\Delta E = h\nu = 9.41 \times 10^{-25} J = 5.77\ \mu eV$. If thermal energy is sufficient, then $KT \sim \Delta E$, or $T \sim \Delta E / k = 0.068\ K$. Yes, the temperature of H I clouds are sufficient to produce this low-energy state.

Problem 7: C&O 12.4

Equation 12.7 reads:

$$\tau_H = 5.2 \times 10^{-23} \frac{N_H}{T \Delta v}. \tag{7}$$

Taking $\tau_H = 0.5$, $T = 100\ K$, and $\Delta v = 10\ km\ s^{-1}$, this equation gives $N_H = 9.6 \times 10^{24}\ m^{-2}$. Since $N_H = n_H d$, we get $d = 31.2\ pc$.