Problem Set 5 Solution

AY 7B

Spring 2012

Problem 25.3

(a) For the Sun, $L_B = 2.3 \times 10^{10}\ L_\odot$. Since $M_{V,Sun} = 4.82$ and $B - V = 0.65$, we find $M_{B,sun} = 5.47$. Thus, from Eq. (3.8), $M_B = M_{B,Sun} - 2.5log_{10}(L_B/L_\odot) = -20.4$.

(b) Using the relation for Sb galaxy, $V_{max} = 184\ km\ s^{-1}$. Observationally, $V_{max} = 250\ km\ s^{-1}$.

Problem 25.8

(a) From Eq. (25.4), $M_B = -21.8$.

(b) According to Eq. (3.6), $B - M_B = 5log_{10}(d/10\ pc)$, or $d = 6.45\ Mpc$.

(c) From Eq. (25.11), $R_{25} = 26.8\ kpc$.

(d) $M_{25} = 6.5 \times 10^{11} M_\odot$ from Eq. (25.10).

(e) From Problem 25.3, $M_{B,Sun} = 5.47$. Then, according to Eq. (3.8), $L_B = 8.1 \times 10^{10} L_\odot$.

(f) $M_{25}/L_B = 8.0$.

Problem 25.9

For Sa, $< B - V >= 0.75$, corresponding to spectral class G8; for Sb, $< B - V >= 0.64$, which corresponds to spectral class G2; and for Sc, $< B - V >= 0.52$, corresponding to F8.

Problem 26.5

Treating the MIlky Way and the LMC as a binary system with a separation of $d_{LMC} = 51\ kpc$, we can use Eq. (18.10) to estimate the tidal radius of the LMC due to the MIlky Way. From Table 24.1, we take $M_1 = 5.4 \times 10^{11} M_\odot$ for the Milky Way within 50 kpc of the center (including its dark matter halo) and $M_2 = 2 \times 10^{10} M_\odot$ for the LMC. Plugging in these values to Eq. (18.10) gives:

$$l_2 = 8.9\ kpc. \tag{1}$$

If the angular diameter of the LMC is $\theta = 460' = 0.134\ rad$, then its linear radius is $r_{LMC} = d_{LMC}\theta/2 = 3.3\ kpc$. Since $r_{LMC} < l_2$, the LMC lies within its tidal radius.

Problem 26.8

With the assumption $r >> a$,

$$\rho(r) \sim \frac{C}{r^2} \ . \tag{2}$$

The enclosed mass, M_r, can be obtained by taking the integral:

$$M_r = \int_0^r \rho(r) 4\pi r^2 \mathrm{d}r = 4\pi C r \ . \tag{3}$$

The radial equation of motion becomes:

$$\frac{d^2 r}{dt^2} = -\frac{GM_r}{r^2} = -\frac{4\pi GC}{r} \ . \tag{4}$$

Multiplying through by $v = dr/dt$, and integrating,

$$\int_0^t v\frac{dv}{dt}dt \ = \ -\int_{r_0}^r \frac{4\pi GC}{r}dr, \tag{5}$$

$$\text{giving } v^2 = 8\pi GC ln(\tfrac{r_0}{r}).$$

Now, take the square root of this expresion and choose the minus sign (since the nebula is collapsing) to get:

$$\frac{dr}{dt} = -(8\pi GC)^{1/2}\left[ln\left(\frac{r_0}{r}\right)\right]^{1/2.} \ . \tag{6}$$

This equation can be integrated:

$$\int_{r_0}^0 \frac{dr}{(ln(r_0/r))^{1/2}} \ == \int_0^{t_{ff}} (8\pi GC)^{1/2} dt \ . \tag{7}$$

We can then solve for t_{ff}, giving:

$$t_{ff} = \frac{r_0}{(8GC)^{1/2}} \ . \tag{8}$$

This shows that the free-fall time is proportional to the radius ($t_{ff} \propto r_0$).

Problem 26.11

(a) From the data in Example 26.2.2, $T_{virial} = 6 \times 10^5\ K$, $\mu = 0.6$, and $n = 5 \times 10^4\ m^{-3}$, implying $\rho = \mu m_H n = 5 \times 10^{-23}\ kg\ m^{-3}$. Then from Eq. (12.14),

$$M_J \sim \left(\frac{5kT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2} = 5.3 \times 10^{11}\ M_\odot\ . \tag{9}$$

(b) Assuming that $T_{virial} \sim 10^4\ K$ and all other values are as given in part (a), then $M_J = 1.1 \times 10^9\ M_\odot$.

(c) Using $T \sim 6 \times 10^5\ K$, Eq. (12.16) gives:

$$R_J = \left(\frac{15kT}{4\pi G\mu m_H \rho_0}\right)^{1/2} = 55\ kpc\ . \tag{10}$$

This value is comparable to the radius of the stellar halo ($R = 50\ kpc$).

Problem 26.13

(a) Taking $M = 3 \times 10^{13}\ M_\odot$ and $R = 300\ kpc$, Kepler's third law implies an orbital period of:

$$P = \frac{4\pi R^{3/2}}{G^{1/2}M^{1/2}} = 1.8 \times 10^{17}\ s = 5.6\ Gyr\ . \tag{11}$$

This is roughly one-third the age of the Milky Way Galaxy.

(b) M87 is not in virial equilibrium. Objects near the outer edges of the galaxy take too long to orbit relative to the age of the universe.