L1 – Special Relativity: I

The speed of Light

SR describes physics at high speeds, approaching the speed of light (c).

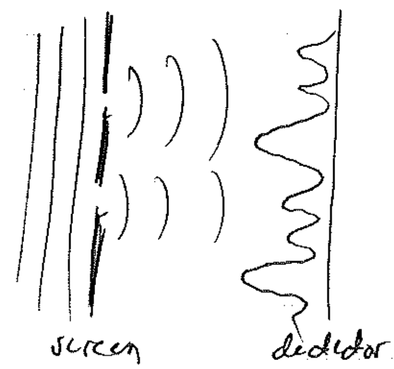
What is special about c?

(2.998×10^8 m/s)

In the 1860's, Maxwell derived c from the laws of E & M (his equations).

Everyone knew about interference, indicating light is a wave phenomenon

So it was assumed that c represents the speed in a medium, called the "ether".



Maxwell also showed that light carries momentum. In fact, he demonstrated that

$$\boxed{E = pc}$$

Relation to Material Objects

In ordinary mechanics, applying a steady force to an object gives it an arbitrarily high speed.

Consider an object falling due to Earth's gravity. The acceleration is $g = 9.8$ m/s².

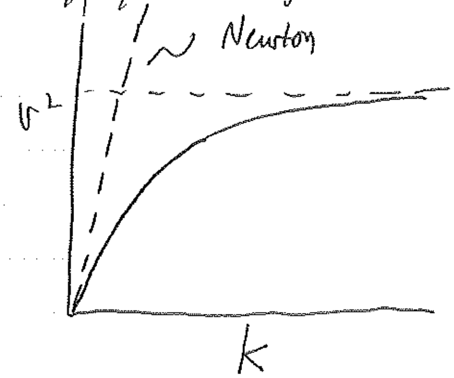
After 1 yr, starting at rest, $v = 3 \times 10^8$ m/s ($= c$)

" 2 yr " " 6×10^8 m/s ($= 2c$) etc

Another way of putting this is: the more work is done on the particle, the higher its speed. $K = \frac{1}{2}mv^2 \longrightarrow v = (2K/m)^{1/2}$

This is not true. One can accelerate electrons using a big potential difference. The kinetic energy gained is just $K = eV$, where V is the voltage drop.

One finds:



> So c is also the ultimate speed obtained by any object!

Constancy of c

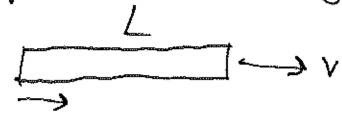
If light is a wave in the ether, its speed should be independent of that of the source (emitter) of the light. This is true.

Example – Unstable particles (π mesons) travel at $> 99.9\%$ of c. The measured speed of the light emitted forward with respect to this motion is exactly c.

But then the apparent speed of light, as detected by an observer moving through the ether ~~[illegible]~~ at speed v, should depend on v. In other words, the measured speed of light should depend on the motion of the receiver.

Example – Light is sent out along a rod of length L, which is moving with V. It bounces and returns.

What is total travel time?



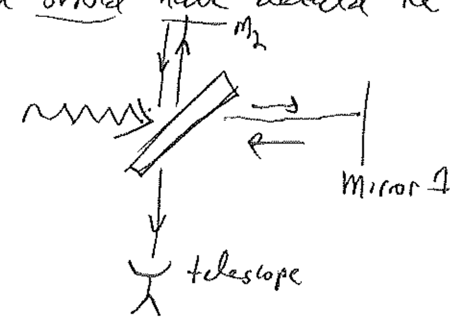
On the way out, (relative) apparent speed is $c - v$. On the way back, it is $c + v$.

So $$t = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2Lc}{c^2 - v^2} \doteq \frac{2L}{c}\left(1 + \frac{v^2}{c^2}\right)$$

If there were no motion in the ether, t would be just $2L/c$.

So there should be a difference $\Delta t \doteq \frac{2L}{c}\left(\frac{v^2}{c^2}\right)$ NOT OBSERVED

For Earth, $v/c \simeq 10^{-4}$, so difference is tiny. Nevertheless, Michaelson + Morley (1887) devised a brilliant experiment that should have detected the difference. They saw none.*



$$\boxed{\text{Light travels at the same speed, regardless of the velocity of the source or of the receiver.}}$$

Einstein's Postulates

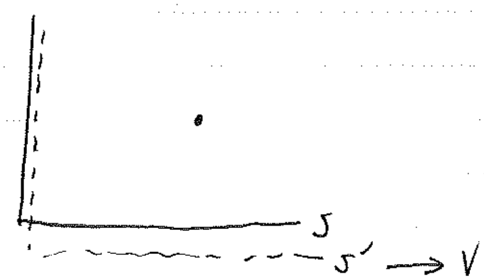
In Newtonian physics (mechanics), it was accepted that the laws of physics ($F = ma$) are the same in all inertial reference frames. Galileo: If an object is dropped from the mast of a moving ship, it still lands at the foot of the mast. In mechanics, one cannot discern the speed of one's reference frame by any mechanical process.

If the speed of light did depend on one's motion w/ respect to the ether, then one could determine the speed of one's reference frame by (non-)mechanical processes. Einstein extended the universality of the laws of physics to include all processes:

① The same laws of physics hold in all inertial reference frames.
② The speed of light always has the same value in all such frames.

Galilean Transformation

Consider two reference frames, S and S'. S' is moving at V with respect to S. Their origins coincide at $t=0$.



An "Event" (e.g., a flash) occurs at (x,y,z,t) according to S-observer. What are its space-time coordinates according to S'?

$$\boxed{\begin{aligned} x' &= x - Vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}}$$

Lorentz Transformation

We need a new transformation that will make c a constant. Consider a spherical wavefront of light emitted at the common origin of S + S' at $t=t'=0$. In S, points along the wavefront obey:

$$x^2 + y^2 + z^2 = c^2t^2. \qquad ①$$

We want also: $$(x')^2 + (y')^2 + (z')^2 = c^2(t')^2 \qquad ②$$

* Try Galilean Transformation: $(x-Vt)^2 + y^2 + z^2 = c^2t^2$ contradicts
(Plugging into ②)

$$①' \quad x^2 - 2xVt + V^2t^2 + y^2 + z^2 = c^2t^2 \qquad ①$$

We suspect the new transformation keeps $y'=y$ and $z'=z$. The trouble is x' + t'!

Try, for simplicity, a transformation that is linear in x and t. It is clear that $t' \neq t$, since there are unwanted terms in ①' involving t.

Next, try: $x' = x - vt\,,\quad y' = y\,,\quad z' = z\,,\quad t' = t + fx$

Plugging into (2),

$$x^2 - 2xvt + v^2t^2 + y^2 + z^2 = c^2t^2 + 2c^2tfx + c^2f^2x^2$$

The terms in xt cancel if we set

$$-2xvt = +2c^2xft \longrightarrow f = \frac{-v}{c^2} \longrightarrow t' = t - \frac{vx}{c^2}$$

We now have:

$$x^2 + v^2t^2 + y^2 + z^2 = c^2t^2 + c^2\frac{v^2}{c^4}x^2$$

$$x^2\left(1 - \frac{v^2}{c^2}\right) + y^2 + z^2 = c^2t^2\left(1 - \frac{v^2}{c^2}\right)$$

So we have extra scale factor $\left(1 - v^2/c^2\right)^{1/2}$ multiplying both x and t.

Finally, try

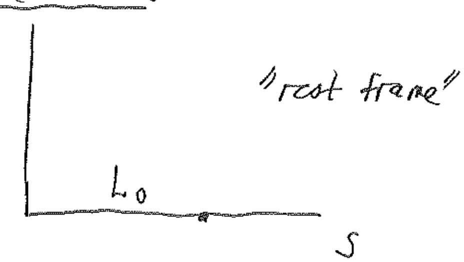
$$\boxed{x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}\,,\quad y' = y\,,\quad z' = z\,,\quad t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - v^2/c^2}}}$$

Plugging into (2), $x^2 + y^2 + z^2 = c^2t^2$ ✓

Notation: $\beta \equiv v/c\,,\quad \gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1-\beta^2}} > 1$ (Lorentz factor) *[so, eg. $t' = \gamma\left(t - \beta\frac{x}{c}\right)$]

Length Contraction

One result of the new transformation is that two events which are simultaneous in one reference frame are not simultaneous in a second, one. This leads to strange consequences. For example, a moving rod shrinks.



Consider two flashes, occurring at the ends of a rod of length L_0, in S. They occur simultaneously:

$$x_1 = 0 \quad t_1 = 0$$
$$x_2 = L_0 \quad t_2 = 0$$

Since the rod is stationary in S, the flash at the right is always a distance L_0 from the left, regardless of when that righthand flash occurs. $\boxed{x_2 - x_1 = L_0 \text{ always}}$

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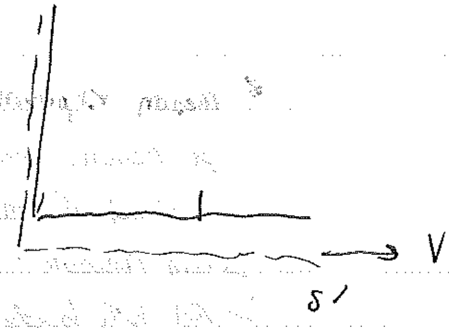
* The inverse transformation is $x = \gamma(x' + \beta ct')$, $y = y'$, $z = z'$, $t = \gamma\left(t' + \frac{\beta x'}{c}\right)$

(5)

Now consider measuring the length of this rod in the moving frame S'! We must make sure that the two flashes occur <u>simultaneously</u> in S'!



So we take $x_1' = 0 \quad t_1' = 0$
$x_2' = L' \quad t_2' = 0$

Applying the inverse Lorentz transformation:

$$x_1 = \gamma(x_1' + Vt_1') = 0$$
$$x_2 = \gamma(x_2' + Vt_2') = \gamma L'$$

It is true that $t_2 \neq t_1$, but, in any event, $x_2 - x_1 = L_0$ ["proper length"]

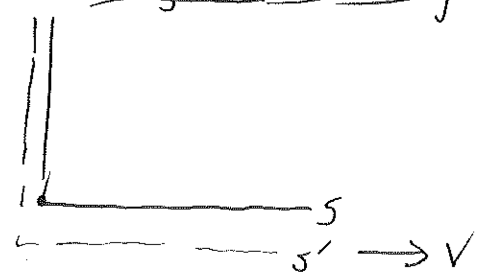
So $L_0 = \gamma L' \longrightarrow \boxed{L' = \frac{L_0}{\gamma}}$ The rod shrinks!

NB: A rod moving perpendicular to its length does <u>not</u> shrink.

Time Dilation

Another strange consequence of the transformation is that <u>moving clocks run slowly</u>.

In S, $x_1 = 0, \; t_1 = 0$
$x_2 = 0, \; t_2 = \tau_0$ "proper time" duration of a tick



In S', $t_1' = \gamma\left(t_1 - \frac{\beta x_1}{c}\right) = 0$

$$t_2' = \gamma\left(t_2 - \frac{\beta x_2}{c}\right) = \gamma\tau_0$$

But $t_2' - t_1' \equiv t'$, the duration of a tick as seen in S'

So $\boxed{t' = \gamma\tau_0}$

Example A π meson ($m = 273\, m_e$) has a "proper lifetime" $\tau = 2.5 \times 10^{-8}$ s
~~So without time dilation~~ Beams can be produced with

$$\beta = 1 - (5 \times 10^{-5}) = 1 - \epsilon, \text{ where } \epsilon \equiv 5 \times 10^{-5}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \approx \frac{1}{\sqrt{2\epsilon}} \qquad (\beta^2 \approx 1 - 2\epsilon; \; 1 - \beta^2 \approx 2\epsilon)$$

$2\epsilon = 1 \times 10^{-4} \longrightarrow \boxed{\gamma = 100}$

Without time dilation, meson would travel $D \approx c\tau = 7$ m before decaying. Actually, it travels $\gamma c\tau = 700$ m!*