L11 – ISM: II. Molecular Clouds

Formation of H_2

Most molecules form in ion-molecule reactions. However, the super-abundant H_2 does not.

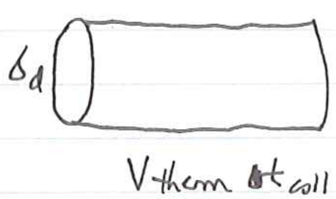
Here, a 3rd body is necessary to absorb the energy of two H atoms binding together. The third body is a dust grain.

The grain's heat capacity is so huge that it easily absorbs the energy without an appreciable rise in temperature.

Let the gas contain n_{HI} atoms per volume. These have thermal velocity V_{therm}. Then a single grain, of geometric cross section σ_d, is struck by an atom, on average, within a time t_{coll}. What is t_{coll}?

t_{coll}

To an H atom, the grain is moving at V_{therm}:



The grain sweeps out a volume $\sigma_d V_{therm} t_{coll}$ over time t_{coll}. The number of H atoms in that volume is (by definition of t_{coll}) – 1.

$$n_{HI}\, \sigma_d\, V_{therm}\, t_{coll} = 1$$

$$\boxed{t_{coll} = (n_{HI}\, \sigma_d\, V_{therm})^{-1}}$$

grain surface exploration

The atom is attracted to the surface by the (weak) Van der Waals force. It explores the surface via QM tunneling.

In a very short time, it comes to rest at a lattice defect, where it forms a stronger bond + sticks.

Within another time t_{coll}, a 2nd atom also explores the surface + finds a binding site adjacent to the first. Only then do the two atoms combine.

The resulting H_2 molecule has no unpaired electrons + pops off easily
[It is in an excited vibrational state — observed!]

The rate of formation of H_2 molecules per volume per time is

$$\mathcal{R}_{H_2} = \tfrac{1}{2}\gamma_H\, n_d\, t_{coll}^{-1}$$
$$= \tfrac{1}{2}\gamma_H\, n_d\, n_{HI}\, \sigma_d\, V_{therm}$$

Here n_d is the grain number density, γ_H is the "sticking probability": the fraction of H atoms striking a grain that recombines $\simeq 0.3$.

self-shielding

None of this happens if UV photons dissociate H_2. But the molecule is "self-shielding." Each dissociation takes away a UV photon, allowing other, interior molecules to survive.
In practice, a depth of $A_V \sim 1$ on the cloud surface is HI, while inside the H is all molecular.

early Universe

In the early Universe, there were no grains, so H_2 did form by gas-phase reactions:

$$H + e^- \rightarrow H^- + h\nu$$
$$H^- + H \rightarrow H_2 + e^-$$

Requires a small degree of ionization. If densities were sufficiently high, the reaction was

$$H + H + H \rightarrow H_2 + H$$

Observed Molecular Clouds

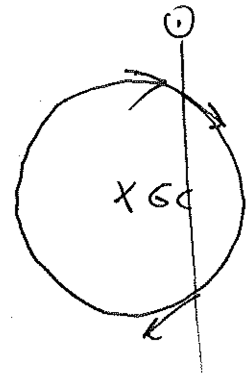
Outside our solar neighborhood, what we see are giant molecular clouds.
These are at the top of a hierarchy —

	L	T	M
GMC's	50 pc	15 K	$10^5\ M_\odot$
[dark clouds / clumps]	2 pc	10 K	10^2–$10^3\ M_\odot$
dense cores	0.1 pc	10 K	10 $M_\odot$

> All OB associations are associated with GMCs. (In fact, that is how they were first identified.)

To map GMCs in the Galaxy, we look at the intensity of CO emission at various Galactic latitudes. The Doppler shift of the line center tells you the distance if you know the "rotation curve" of the Galaxy:

But note the distance ambiguity.



$\boxed{\text{Show Fig 3.2 Galactic distribution of GMCs}}$ handout

We find that GMCs trace spiral arms

The full intensity, integrated over all v, gives you the mass of CO. Since CO is a fixed fraction (by mass) of H_2 ($\sim 10^{-5}$), this is the chief method for determining the masses of GMCs and also of the H_2 content of external galaxies.

In more detail, astronomers observe I_ν (specific intensity $\frac{\text{erg}}{\text{cm}^2\,\text{s steradian Hz}}$) but call it

$$T_B \equiv \frac{c^2 I_\nu}{2\nu^2 k_B}$$

(from the Rayleigh-Jeans limit of the BB formula for I_ν)

They observe $\boxed{T_A(V_r)}$ where $V_r/c = \frac{\nu_0 - \nu}{c}$

X factor

When observing far off clouds, use the "X factor" to convert the CO column density to an H column density:

$$\boxed{N_H = X \int T_A \, dV_r}$$

a complication: CO is optically thick

Virial Theorem Analysis

From Euler's equation:

$$\rho \frac{D\vec{u}}{Dt} = -\vec{\nabla} p - \rho \vec{\nabla}\phi_g + \frac{1}{c}\vec{J}\times\vec{B} \qquad \left[\frac{D\vec{u}}{Dt} = \frac{\partial u}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u}\right]$$

Use Ampère's law: $\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J}$ (neglecting displacement current)

$$\rho \frac{D\vec{u}}{Dt} = -\vec{\nabla} p - \rho\vec{\nabla}\phi_g + \underbrace{\frac{1}{4\pi}(\vec{B}\cdot\vec{\nabla})\vec{B}}_{\text{magnetic tension}} - \underbrace{\frac{1}{8\pi}\vec{\nabla}|\vec{B}|^2}_{\text{magnetic pressure}}$$

Dot Euler with $\vec{r}\cdot$ and integrate to find

$$\boxed{\frac{1}{2}\ddot{I} = 2\mathcal{T} + 2U + W + \mathcal{M}}$$

$$I \equiv \int \rho |\vec{r}|^2 d^3x \qquad \underbrace{U \equiv \frac{3}{2}\int P\, d^3x}_{\text{thermal energy}} \qquad \underbrace{W \equiv \frac{1}{2}\int \rho\,\phi_g\, d^3x}_{\text{gravitational}}$$

Application: Free fall of GMC

Ignore everything but $\ddot{I}$ and W

$$\frac{1}{2}\ddot{I} = W$$

Roughly —

$$\frac{I}{t_{ff}^2} \sim \frac{GM^2}{R} \qquad \text{since } I \simeq MR^2$$

$$\boxed{t_{ff} = \left(\frac{R^3}{GM}\right)^{1/2} = \frac{1}{\sqrt{G\rho}}} \qquad \left[\text{conventional definition is } t_{ff} = \sqrt{\frac{3\pi}{32 G\rho}}\right]$$

NB — depends only on density!

If use $M = 10^5\, M_\odot$, $R = 25$ pc, $n = 30$ cm^{-3}

$$\longrightarrow \rho = 1\times10^{-22}\ \text{gm/cm}^3 \longrightarrow t_{ff} = 1\times10^7\ \text{yr}$$

More realistically, GMC's are collections of clumps, & each clump has $n \simeq 10^3$ cm^{-3} $\longrightarrow$ $\rho = 3\times10^{-21}$ gm/cm^3

$$\longrightarrow \boxed{t_{ff} = 2\times10^6\ \text{yr}}$$

Galactic Star Formation Rate

The total H_2 mass in the Galaxy is $2 \times 10^9\ M_\odot$.
Suppose all this mass is undergoing free-fall collapse to form stars.

Then the global SF rate in the Galaxy would be $\dot{M}_* = \frac{2 \times 10^9\ M_\odot}{2 \times 10^6\ \text{yr}} \approx 10^3\ M_\odot/\text{yr}$

Observed SF rate $= 4\ M_\odot\ \text{yr}^{-1}$!! "Star formation is inefficient"

Two possible reasons—
(i) Clouds are indeed collapsing, but only a small fraction form stars.
(ii) " " NOT " , but live longer than t_{ff}

The 2nd reason is definitely true, but (i) is also

$\boxed{\text{Show Fig 3.8 — lifetime of GMCs}}$ handout

Relative Importance of Virial Terms

For $\ddot{I} = 0$, $\quad 0 = 2\mathcal{T} + 2\mathcal{U} + \mathcal{W} + \mathcal{M}$

> $\frac{\mathcal{U}}{|\mathcal{W}|}$: Use $\mathcal{U} \doteq \frac{3}{2}\frac{M \mathcal{R} T}{\mu}$ $\quad |\mathcal{W}| \doteq \frac{GM^2}{R}$ $\quad \boxed{\frac{\mathcal{U}}{|\mathcal{W}|} = 3 \times 10^{-3}}$

GMC's are not supported by thermal pressure.

> $\frac{\mathcal{M}}{|\mathcal{W}|}$: Use $\mathcal{M} = \frac{4}{3}\pi R^3 \frac{B^2}{8\pi} = \frac{B^2 R^3}{6}$ $\quad B \simeq 20\ \mu G$

$\boxed{\frac{\mathcal{M}}{|\mathcal{W}|} \approx 0.9}$

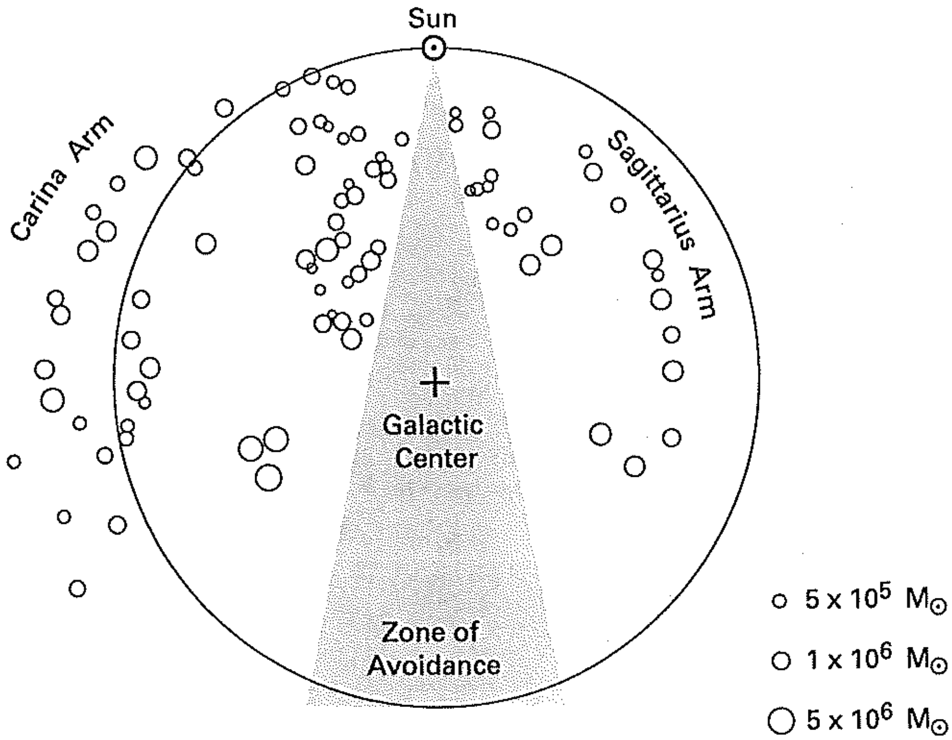
Support by MHD waves, not static field

> $\frac{\mathcal{T}}{|\mathcal{W}|}$ $\quad \mathcal{T} = \frac{1}{2} M \Delta V^2$ ← from width of CO

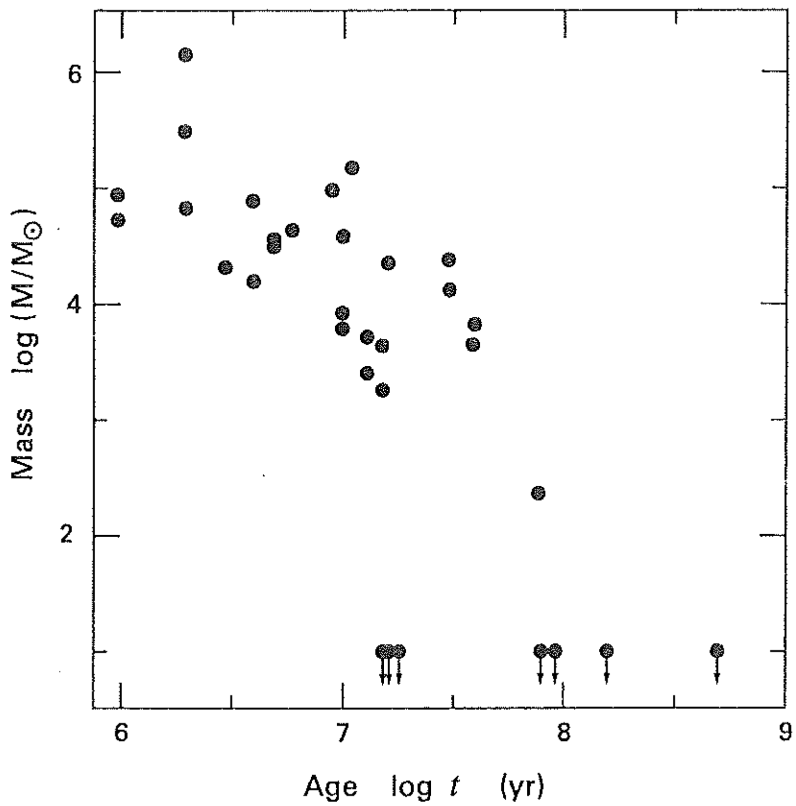
$\Delta V \simeq 4\ \text{km s}^{-1}$ $\quad \boxed{\frac{\mathcal{T}}{|\mathcal{W}|} \approx 0.5}$

Roughly $\Delta V \sim \sqrt{\frac{GM}{R}}$ ("virial velocity").

Physical picture— GMC's consist of clumps that are moving in the gravitational well of the entire complex. There is equipartition between $\mathcal{T}$ and $\mathcal{M}$, since gas is exciting MHD waves.



Sun
Carina Arm
Sagittarius Arm
Galactic Center
Zone of Avoidance
5 x 10^5 M⊙
1 x 10^6 M⊙
5 x 10^6 M⊙



6
4
2
Mass log (M/M$_\odot$)
6
7
8
9
Age log t (yr)