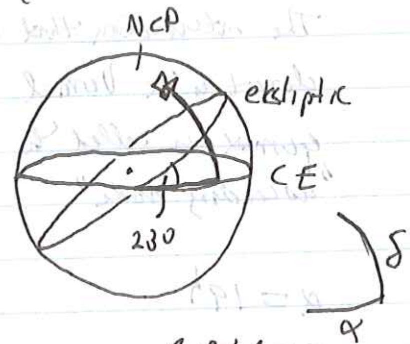
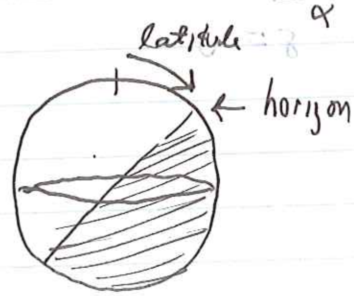
L13 - The Milky Way: Motion of Stars

Coordinate Systems (Earth-centered)

Within the celestial sphere, stars turn around the north celestial pole (NCP).

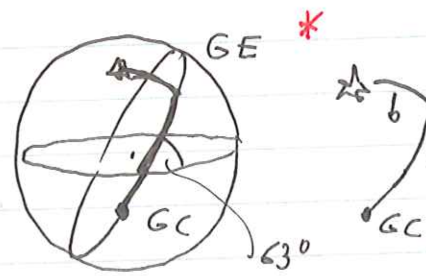


The local horizon is a plane whose angle with respect to the NCP ~~depends on~~ is just the observer's latitude



The path followed by the sun is the ecliptic. It forms an angle of 23° with respect to the ~~eq~~ celestial equator.

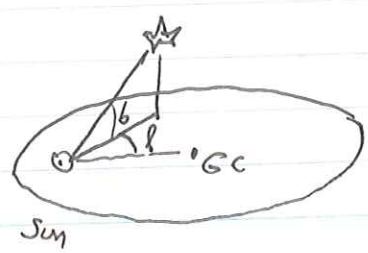
In the equatorial system, the star's position is right ascension (α – in hours, minutes, seconds) and declination (δ – in degrees, arcminutes, arcseconds). The zero point for α is the vernal equinox (in Aries) where the ecliptic intersects the celestial equator.



In the galactic coordinate system, we place objects with respect to the plane of the Milky Way. This plane is tilted 63° from the celestial equator.

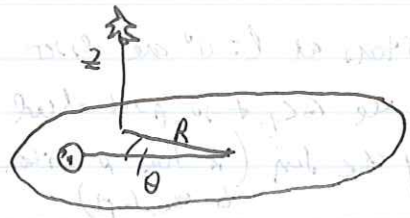
We measure the longitude (l) with respect to the Galactic center (in Sagittarius, southern hemisphere). Also measure the latitude (b) north or south of the galactic equator (GE). Both l and b are in degrees, arcmin, and arcsec.

If we reposition the Galactic plane horizontally and put the galactic center the middle, l and b are more pictured:



Galacto-centric Coordinates and the LSR

A more natural reference frame is one whose origin is at the Galactic center.



With respect to that frame, the motion of stars is conventionally denoted as

$$\Pi \equiv \frac{dR}{dt} \qquad \Theta \equiv R\frac{d\theta}{dt} \qquad Z \equiv \frac{dz}{dt}$$

The local standard of rest (LSR) is instantaneously centered on the Sun and moving in a perfectly circular orbit about the Galactic center.

The orbital speed of the LSR is $\boxed{\Theta_0 = 220 \text{ km s}^{-1}}$

Using $R_0 = 8$ kpc, the period of the Sun about the Galactic center is 230 Myr.

Differential Rotation of Stars

The velocity Θ varies with R. So does $d\theta/dt \equiv \Omega$. This Galactic differential rotation is characterized locally by the Oort constants:

Shear: deviation from rigid rotation

$$A \equiv -\frac{1}{2}\left[\left.\frac{d\Theta}{dR}\right|_0 - \frac{\Theta_0}{R_0}\right]$$

(ang. momentum) vorticity gradient

$$B \equiv -\frac{1}{2}\left[\left.\frac{d\Theta}{dR}\right|_0 + \frac{\Theta_0}{R_0}\right] = -\frac{1}{2}\frac{1}{R_0}\left.\frac{d(R\Theta)}{dR}\right|_0$$

both have units of Ω

These constants are useful for describing the motion of stars relative to the Sun.

We will show that $V_r = A\, d \sin 2l$

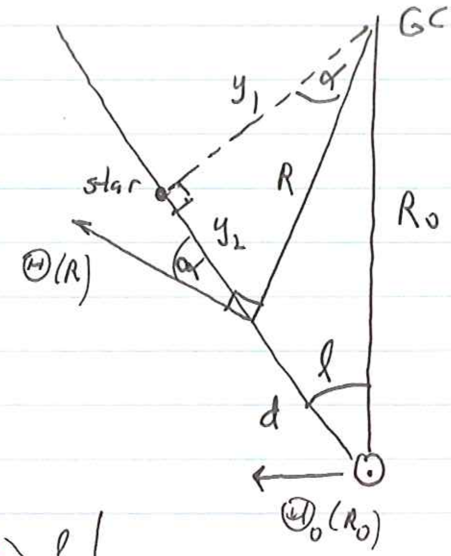
d = distance to *

$V_t = A\, d \cos 2l + B\, d$

* History - By 1900, it was known that proper motion ($\propto V_t$) had a $\cos 2l$ dependence. Oort developed the theory in 1927.

Obtain A & B by measuring (V_r, V_t) for stars of known d & l.

The star is located at longitude ℓ and distance d from the Sun.

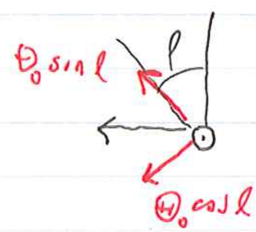


Problem: Find V_r and V_t (relative velocities) as functions of ℓ and d.

Let $\Theta(R)$ be the orbital velocity of the stars relative to the Galactic center. Then –

$$V_r = \Theta \cos\alpha - \Theta_0 \sin\ell$$

$$V_t = \Theta \sin\alpha - \Theta_0 \cos\ell$$



Recall that $\Omega(R) = \dfrac{\Theta(R)}{R} \rightarrow$

$$V_r = \Omega R \cos\alpha - \Omega_0 R_0 \sin\ell$$

$$V_t = \Omega R \sin\alpha - \Omega_0 R_0 \cos\ell$$

Note that

$$y_1 = R\cos\alpha = R_0 \sin\ell$$
$$y_2 = R\sin\alpha = R_0 \cos\ell - d$$

so –

$$V_r = \Omega R_0 \sin\ell - \Omega_0 R_0 \sin\ell = (\Omega - \Omega_0) R_0 \sin\ell \leftarrow$$ [we will later use this general expression]

$$V_t = \Omega R_0 \cos\ell - \Omega d - \Omega_0 R_0 \cos\ell = (\Omega - \Omega_0) R_0 \cos\ell - \Omega d$$

Now consider stars close to us ($d \ll R_0$), which are the only ones for which we can measure d.

$$\Omega(R) = \Omega_0(R_0) + \left.\frac{d\Omega}{dR}\right|_{R_0} (R - R_0) + \ldots$$

$$\Omega - \Omega_0 = \frac{d\Omega}{dR}(R - R_0)$$

$$\boxed{\begin{aligned} V_r &= \frac{d\Omega}{dR}(R - R_0) R_0 \sin\ell \\ V_t &= \frac{d\Omega}{dR}(R - R_0) R_0 \cos\ell - \Omega d \end{aligned}}$$

We may recast these in terms of Θ. Use $\Omega = \dfrac{\Theta}{R}$

$$\frac{d\Omega}{dR} = -\frac{\Theta}{R^2} + \frac{1}{R}\frac{d\Theta}{dR}$$

$$V_r = \left(-\frac{\Theta_0}{R_0^2} + \frac{1}{R_0}\frac{d\Theta}{dR}\right)(R-R_0)R_0 \sin l$$

$$\boxed{V_r = \left(\frac{d\Theta}{dR} - \frac{\Theta_0}{R_0}\right)(R-R_0)\sin l}$$

$$V_t = \left(\frac{1}{R_0}\frac{d\Theta}{dR} - \frac{\Theta}{R_0^2}\right)R_0(R-R_0)\cos l - \Omega_0 d$$

$$\boxed{V_t = \left(\frac{d\Theta}{dR} - \frac{\Theta_0}{R_0}\right)(R-R_0)\cos l - \Omega_0 d}$$

Since $d << R$ $\quad R_0 - R \simeq d\cos l$

[The exact equality is $R_0 = d\cos l + R\cos\beta$, but $\beta << 1$]

$$V_r = \left(\frac{d\Theta}{dR} - \frac{\Theta_0}{R_0}\right)(-d\sin l\cos l) = -\frac{1}{2}\left(\frac{d\Theta}{dR} - \frac{\Theta_0}{R_0}\right)d\sin 2l$$

$$V_t = \left(\frac{d\Theta}{dR} - \frac{\Theta_0}{R_0}\right)(-d\cos^2 l) - \Omega_0 d$$

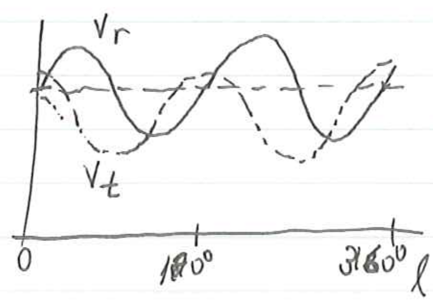
$$\cos 2l = \cos^2 l - \sin^2 l = 2\cos^2 l - 1 \rightarrow \cos^2 l = \frac{1+\cos 2l}{2}$$

$$= -\frac{1}{2}\left(\frac{d\Theta}{dR} - \frac{\Theta_0}{R_0}\right)d\cos 2l - \left(\frac{d\Theta}{dR} - \frac{\Theta_0}{R_0}\right)\frac{d}{2} - \frac{\Theta_0}{R_0}d$$

$$\boxed{V_t = -\frac{1}{2}\left(\frac{d\Theta}{dR} - \frac{\Theta_0}{R_0}\right)d\cos 2l - \frac{1}{2}\left(\frac{d\Theta}{dR} + \frac{\Theta_0}{R_0}\right)d}$$

—or—

$$\boxed{\begin{aligned} V_r &= A\,d\sin 2l \\ V_t &= A\,d\cos 2l + B\,d \end{aligned}}$$



for $l = 0°$, have only V_t

for $l = 90°$, have only V_t

again, since star shares the same $\Omega\ (=\Omega_0)$.

Once we determine A + B empirically, use $\Omega_0 = A - B$ *

$\frac{d\Omega_0}{dR} = -(A+B)$

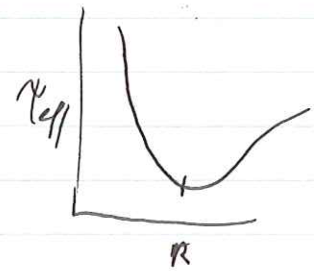
Current values: $A \doteq 14.8 \text{ km s}^{-1}\text{ kpc}^{-1}$

$B = -12.4 \text{ km s}^{-1}\text{ kpc}^{-1}$

Alternate expressions:

$$\boxed{\begin{aligned} A &= -\frac{1}{2}\left(R\frac{d\Omega}{dR}\right)_0 \\ B &= -\left(\frac{1}{2}R\frac{d\Omega}{dR} + \Omega\right)_0 \end{aligned}}$$

Epicyclic Motion of the Sun



Effective potential

$$\Psi_{eff} = \Psi + \frac{l^2}{2R^2}$$

where Ψ is not keplerian

Eqtn of motion: $\frac{d^2R}{dt^2} = -\frac{d\Psi_{eff}}{dR}$

R_0 found from $\frac{d\Psi_{eff}}{dR} = 0 \longrightarrow \left(\frac{d\Psi}{dR}\right)_{R_0} = \frac{l^2}{R_0^3}$

Let $x \equiv R - R_0$ & consider motion for small x.

$$\Psi_{eff} \doteq (\Psi_{eff})_0 + \frac{1}{2}\left(\frac{d^2\Psi_{eff}}{dR^2}\right)_0 x^2$$

$$\frac{d\Psi_{eff}}{dR} \doteq \left(\frac{d^2\Psi_{eff}}{dR^2}\right)_0 x$$

$\boxed{\ddot{x} = -\kappa^2 x}$ harmonic oscillator , $\boxed{\kappa^2 = \left(\frac{d^2\Psi_{eff}}{dR^2}\right)_0 \text{ epicyclic frequency}}$

$$\kappa^2 = \frac{d^2\Psi}{dR^2} + \frac{3l^2}{R_0^4}$$

As the Sun moves in & out, it maintains a nearly circular orbit * $\Omega^2 R \doteq \frac{d\Psi}{dR}$ Also $l = \Omega_0 R^2$

$$\boxed{\kappa^2 = \frac{d}{dR}(\Omega^2 R) + 3\Omega^2}$$

$$= R\frac{d\Omega^2}{dR} + 4\Omega^2 = 2R\Omega\frac{d\Omega}{dR} + 4\Omega^2 = -4\Omega \times \overbrace{\left(-\frac{1}{2}R\frac{d\Omega}{dR} - \Omega\right)}^{B}$$

$\boxed{\kappa^2 = -4\Omega B}$ (Thus $B<0$) $\quad \frac{\kappa^2}{\Omega^2} = \frac{-4B}{\Omega} = \frac{-4B}{A-B} \longrightarrow \frac{\kappa}{\Omega} = 2\sqrt{\frac{-B}{A-B}} = 1.4$

Sun moves in & out 1.4 times as it completes one revolution about GC.