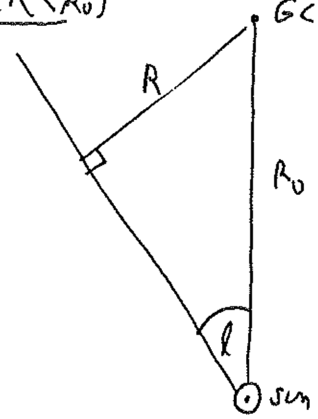
L14 - Rotation Curves

We begin with methods for finding $\Theta(R)$ within the Milky Way

Tangent Point Method ($R<R_0$)



At a fixed l, observed stars have a range of V_r-values.

Consider the star whose orbital velocity is along our line of sight. Its distance R to the G center is the least. Assuming $\Theta(R)$ is a decreasing function of R, the Θ for this star is the maximum.

Now V_r in this case reflects the full Θ (no projection). So this star also has the highest V_r.

The distance R in this case is $R_{min} = R_0 \sin l$. Thus

$$V_{r,max} = \Theta(R_0 \sin l) - \Theta_0(R_0) \sin l$$

Method: At a fixed l, find $V_{r,max}$ observationally. Assume Θ_0 is known ($= 220$ km s^{-1}). Then

$$\Theta(R) = V_{r,max} + \Theta_0 \sin l$$

where $R = R_0 \sin l$

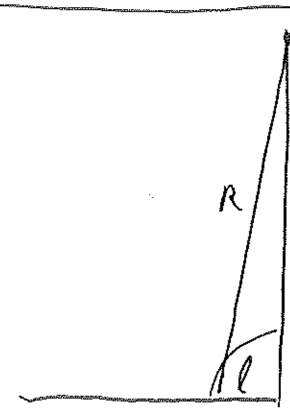
In practice, this method has been more often used with HI gas, not stars. Here, the Doppler shift of the 21-cm line gives V_r. The beauty of the method is that we don't need to know the distance d to the HI cloud — rarely known!

Aside: Determination of R_0

A variant of the method is used to find R_0. Suppose $l \lesssim 90°$. Then $d << R_0$, & we can write

$$\Theta(R_{min}) \doteq \Theta_0(R_0) + \left|\frac{d\Theta}{dR}\right|_{R_0} (R_{min} - R_0)$$

$$= \Theta_0(R_0) + \left|\frac{d\Theta}{dR}\right|_{R_0} R_0 (\sin l - 1)$$



Thus $V_{r,max} = \Theta(\overset{R_{min}}{R_0 \sin l}) - \Theta_0(R_0)\sin l$ * Alternative derivation

$$= \Theta_0(R_0) + \left|\frac{d\Theta}{dR}\right|_0 R_0(\sin l - 1) - \Theta_0(R_0)\sin l$$

$$= \left[\Theta - \left|\frac{d\Theta}{dR}\right|_0 R_0\right](1-\sin l) \qquad \text{But } A \equiv -\frac{1}{2}\left[\left|\frac{d\Theta}{dR}\right|_0 - \frac{\Theta}{R_0}\right]$$

$$\boxed{V_{r,max} = 2AR_0(1-\sin l)}$$ * knowing A, measurement of $V_{r,max}$ vs l gives R_0.

Cluster method ($R > R_0$)

Here, there is no unique orbit that has a maximum V_r. Tangent method fails. However, the HI gas does follow the expected pattern at all l-values. Recall that:

$$\boxed{V_r =} (\Omega - \Omega_0) R_0 \sin l = \boxed{R_0 \sin l \left(\frac{\Theta}{R} - \frac{\Theta_0}{R_0}\right)}$$

GC

If the rotation were rigid, $\frac{\Theta}{R} = \frac{\Theta_0}{R_0}$, & V_r would be 0 at all l. In fact, Θ/R drops with increasing R.

> For $0 < l < 90°$, $V_r > 0$ for nearby objects $\left(\frac{\Theta}{R} > \frac{\Theta_0}{R_0}\right)$, but $V_r < 0$ for objects on the other side of the G center, for which $R > R_0$, so that $\Theta/R < \Theta/R_0$. So in this l-range, should get both + and − V_r.

> For $90° < l < 180°$, all objects have $\Theta/R < \Theta/R_0$, and $\boxed{V_r < 0.}$

> For $180° < l < 270°$, all objects have $\Theta/R < \Theta_0/R_0$, but $\sin l < 0 \rightarrow \boxed{V_r > 0.}$

> For $270° < l < 360°$, we again have a mix of (+) and (−) V_r-values.

The pattern of HI emission follows this expectation

show Fig 2.18 in Sparke & Gallagher

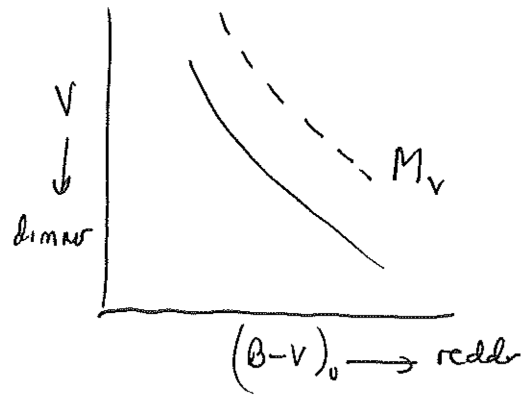
It is easier to find d to a star cluster than an individual star.

If we can find d, then knowing both R_0 and l gives R.

Measuring V_r then gives Θ via

$$\Theta = R\left[\frac{V_r}{R_0 \sin l} + \frac{\Theta_0}{R_0}\right]$$

Find d via <u>spectroscopic parallax</u>



(a) Record the cm $(V, B-V)$ diagram of the <u>main sequence</u> of the cluster (usually OB association)

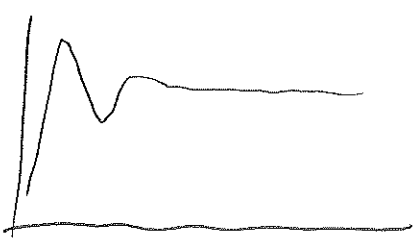
(b) From <u>spectra</u> of stars, replace $B-V$ by $(B-V)_0$.

(c) Compare with standard main-sequence: M_V vs. $(B-V)_0$.

(d) The difference $m_V - M_V$ is the distance modulus: $5\log\left(\frac{d}{10\ \mathrm{pc}}\right)$.

<u>Blitz (1979)</u> used this to get d to OB associations. Each was part of a <u>giant molecular cloud</u>. The 2.6 mm CO line (unlike stellar lines) is narrow ($\Delta V \sim 2$ km/s), so one can find V_r very accurately. Thus found $\Theta(R)$.

The modern result is



Θ tends to a constant (flat) value for $R \gg R_0$.

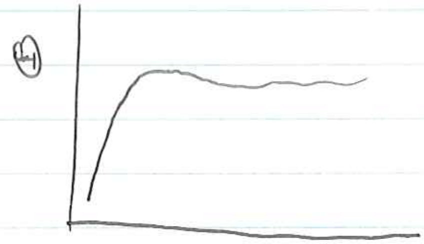
$\boxed{\text{show Fig. 3 from Clemens 1985, ApJ, } \underline{295}\text{, 422}}$

<u>Other Spiral Galaxies</u>

In the 1970's, Rubin and colleagues looked at HI gas in relatively nearby (resolved) galaxies. Measured V_r as a function of angle from the center.

Knowing inclination angle gives $\Theta(R)$.

Found similar pattern to Milky Way.



Near the center, $\Theta \propto R$: rigid rotation

After some fluctuation, $\Theta(R)$ flattens out.
"Flat rotation curves"

Theoretical Expectation

- keplerian – If most of the mass were at the center $(R=0)$, $\boxed{\Theta \propto \frac{1}{\sqrt{R}}}$
- Our galaxy: photometric reconstruction

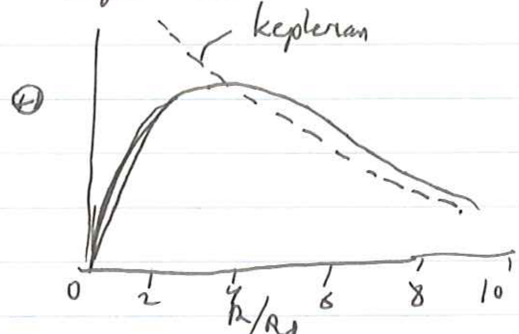
Suppose that all of the Galactic mass were in stars. We could then find the surface mass density $\Sigma(R)$ and hence $\Theta(R)$.

One first finds the luminosity density of stars. Assuming a certain mass-to-light ratio, one converts this to a stellar mass density. For example, we quoted earlier that $\langle M/L \rangle = 3(M_\odot/L_\odot)$ in the Galactic disk.

One can find $\Theta(R)$ exactly for any $\Sigma(R)$ by first finding the gravitational potential $\psi(R)$ and then using

$$\frac{\Theta^2}{R} = \frac{d\psi}{dR}$$

[The ~~full~~ expression for $\psi(R)$ is an integral over $\Sigma(R)$ weighted by the Bessel fn $J_0(kR)$]
The traditional practice was to model $\Sigma(R)$ as an exponential:
$\Sigma = \Sigma_0 e^{-R/R_d}$. The rotation curve is then:



$\Theta(R)$ $\propto R^{1/2}$ ~~is "rigid rotation"~~ in the inner region;*
then reaches a maximum, & finally falls in the keplerian fashion.

There is a qualitative disagreement with the observed $\Theta(R)$.

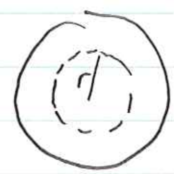
$\Theta^2/R = d\psi/dR =$
* In this case, $M(R) \propto R$ for $R << R_d$. So $\frac{M}{R^2} \sim R^0$

• Spherical mass models

Suppose we make the radical assumption that most galactic mass is not in the disk, but in an unseen spherical component – the dark matter halo.

rigid rotation

It is then easy to reproduce the interior rigid rotation. Let the mass density ρ of the spherical component be ρ_0 near the center. Within the sphere, only the interior mass $M(r)$ exerts gravity. Thus,



$$V^2 = \Theta^2 = \frac{GM(r)}{r} \qquad M(r) = \frac{4\pi}{3}\rho_0 r^3$$

$$\Theta^2 = \frac{4\pi}{3}\rho_0 \frac{G r^3}{r} = \frac{4\pi G \rho_0}{3} r^2$$

This is rigid rotation, with $\Omega^2 = \frac{4\pi G\rho_0}{3}$

Flat $\Theta(r)$

More generally, let $\rho(r) = \rho_0 \left(\frac{r_0}{r}\right)^\alpha \quad \alpha > 0$

Now the mass interior to r is

$$M(r) = \int_0^r 4\pi r^2 \rho \, dr = \frac{4\pi \rho_0 r_0^\alpha}{3-\alpha} r^{3-\alpha}$$

[For $\alpha > 3$, $M(r)$ diverges, so let $\alpha < 3$]

We now find $\Theta^2 = \frac{GM(r)}{r} = \frac{4\pi G \rho_0 r_0^\alpha}{3-\alpha} r^{2-\alpha}$

Since $\Theta^2 \approx$ constant, $\boxed{\alpha \approx 2}$. More specifically, $M(r) = \frac{\Theta^2 r}{G}$ differentiating

$$\frac{dM}{dr} = 4\pi r^2 \rho = \frac{\Theta^2}{G}$$

(Here Θ is the limiting value) $\boxed{\rho = \frac{\Theta^2}{4\pi G r^2}}$ "singular isothermal sphere"

Caveat – Such a density will not give interior rigid rotation, since ρ is not tending to a constant as $r \to 0$. Equally seriously, $M(r)$ diverges as $r \to \infty$.
A modification that addresses the 1st point is

$$\rho(r) = \frac{\rho_0}{1 + (r/a)^2}$$

[But $M(r)$ still diverges]

From N-body simulations, the NFW profile [Navarro, Frenk, & White 1998] is

$$\rho(r) = \frac{\rho_0}{\left(\frac{r}{a}\right)\left(1+\frac{r}{a}\right)^2}$$

$\rho \propto r^{-1}$ for $r \ll a$
$\rho \propto r^{-3}$ for $r \gg a$
In between, $\rho \propto r^{-2}$ roughly

But $M(r)$ still diverges!