L15 - Dark Matter

Evidence from Galaxy Velocities

Coma — The best case is the Coma Cluster: a rich cluster, containing several thousand galaxies; $D = 100$ Mpc.* It's (visible) core radius is 0.2 Mpc. In this central region are several giant elliptical galaxies

Zwicky — Fritz Zwicky (1933) first inferred the presence of dark matter, using Coma. He had V_r - measurements for 7(!) galaxies. They differed from the mean by ~700 km/s

Zwicky made a crude estimate of the cluster radius R & used the virial theorem to get the cluster mass:

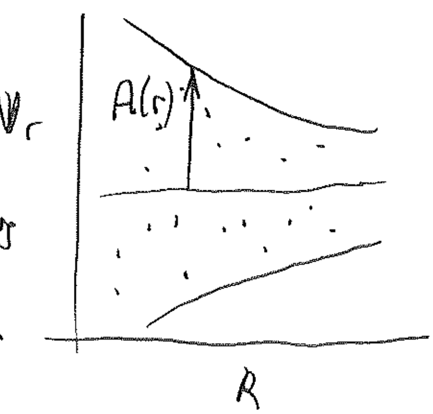
$$\frac{GM}{R} \approx V^2 \longrightarrow M \approx \frac{RV^2}{G}$$

He found $\langle M/L \rangle$ this way and compared it to $\langle M/L \rangle$ in the central region of spiral galaxies. He found

$$\langle M/L \rangle_{\text{Coma}} \approx 400 \times \langle M/L \rangle_{\text{spirals}}$$

He concluded that most matter must be dark.

~~Recent~~ recent — Modern measurements (eg Geller et al 1999, ApJ 517, L20) find redshifts for over 1000 galaxies.



V_r is plotted against radius from the cluster center.
One finds the envelope ("~~caust~~ caustics") separating the cluster from background galaxies
The amplitude $A(r)$ is ½ the caustic separation.
One can show: $GM(r) = \frac{1}{2}\int_0^r A^2(r')\,dr'$

see Kaiser 1987, MNRAS, 227, 1

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* It's redshift is small. If $H = 70$ km/s.Mpc, $V_r = 7\times10^3$ km/s, so
$$z = \frac{V_r}{c} = \frac{7\times10^3}{3\times10^5} = 2\times10^{-2}$$

Geller et al (1999) found that $M(r) = 2\times10^{15}\ M_\odot$ out to 10 Mpc.
This is at least 10 times the mass in galaxies $(7\times10^{13}\ M_\odot)$.

> Moreover, the shape of $M(r)$ is consistent with the NFW profile.

Evidence from X-Rays

Both clusters of galaxies + individual galaxies can emit X-rays. If the X-ray gas is in hydrostatic equilibrium*, then one can infer the gravitational potential, and hence $M(r)$. We have

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}$$

Here, P and ρ refer to the hot gas. The quantity $M(r)$ is the total mass up to radius r. In other words, $M(r) \gg \int 4\pi r^2 \rho\, dr$.

$$\frac{r}{P}\frac{dP}{dr} = \frac{d\ln P}{d\ln r} = -\frac{GM(r)}{r^2}\frac{\rho r}{P} = -\frac{G\mu m_H}{k_B T r} M(r)$$

where I used $P = \frac{\rho k_B T}{\mu m_H}$

But we also have

$$\frac{d\ln P}{d\ln r} = \frac{d\ln\rho}{d\ln r} + \frac{d\ln T}{d\ln r}.$$

We find:

$$\boxed{M(r) = \frac{k_B T r}{G\mu m_H}\left[-\frac{d\ln\rho}{d\ln r} - \frac{d\ln T}{d\ln r}\right]}$$

Gas is optically thin, so luminosity density gives $\rho(r)$ [emission $\propto \rho^2$]
Need a resolved X-ray spectrum to get $T(r)$ — harder to obtain, so often assume isothermality.

Coma

Applying this to the Coma cluster, Cowie et al 1987 found

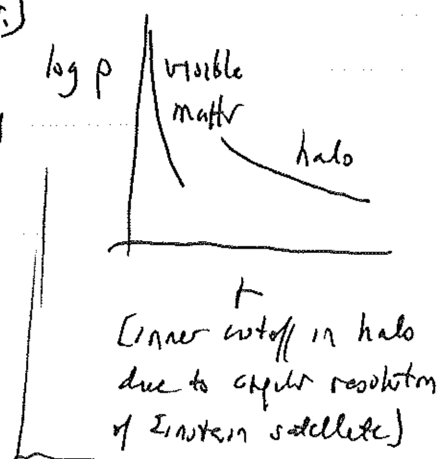
$$\left\langle\frac{M}{L}\right\rangle_{\text{Coma}} = 200\left\langle\frac{M}{L}\right\rangle_\odot$$

* The cooling time of the sparse, hot gas is extremely long.

M87 The method has also been used in $\boxed{M87}$, a giant elliptical galaxy in the core of the Virgo cluster, $\Delta \simeq 15$ Mpc. [and by far the most luminous X-ray source in Virgo cluster]

Applying the method, Fabricant et al (1980)* found that $M(r)$ rises linearly out to $r = 300$ kpc, w/ $M = 3 \times 10^{13} M_\odot$ at that point [In contrast, the core radius of visible matter is 0.8 kpc!]

In comparison $M_{Lum+gas} = 2 \times 10^{12} M_\odot$. [By the way, this huge halo could be related to M87 being at the center of a galaxy cluster.]



Theoretical Evidence: Disk Stability

Ostriker & Peebles (1973) -

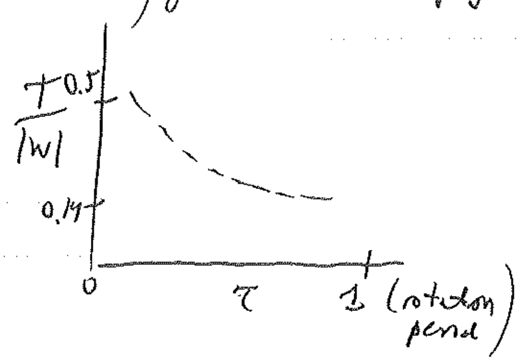
Our galaxy and others are dynamically "cold": random motions are small compared to $\Theta(r)$.

If we idealize the galaxy as a rotating "fluid", then $|T/W|$ is close to its maximum value of 0.5 (from the virial theorem).

Maclaurin (uniform-density) spheroids become secularly unstable for $T/|W| \gtrsim 0.14$. They form bars (Jacobi ellipsoids), since for a given angular momentum L, the energy $E = L^2/2I$ decreases as I increases.

They modeled a galaxy as N (200-500) points, mutually interacting and embedded in a rigid, spherically symmetric "halo." The points were given enough random speed that they were stable against small-scale gravitational clumping.

With $M_H = 0$, $T/|W|$ quickly declined toward 0.14. The galaxy formed a $\boxed{\text{bar}}$. This then dissolved and the structure was a fat "disk" in which $T/|W| \sim 0.14$



Fabricant et al 1980, ApJ, 241, 552

The halo had $M(r) \propto r$. For $M_H \gtrsim M_{disk}$, $T/|W|$ starts out small [the halo has no rotational K.E.] and then just coasts toward $T/|W| \sim 0.14$ without forming a bar.

It thus appears that a massive halo is necessary to prevent any spiral galaxy from (quickly!) developing a bar.

The Solar Neighborhood: Oort Limit

Recall our discussion of the vertical equilibrium of HI gas. We used

$$\frac{1}{\rho}\frac{dP}{dz} = -\frac{d\Phi_g}{dz} \qquad \text{Here } P = \rho c^2$$

We can apply the same equation to the vertical motion of stars:

$$\frac{1}{n_*}\frac{d}{dz}\left(n_* c_z^2\right) = -\frac{d\Phi_g}{dz} \qquad c_z = \text{vertical random speed}$$

But Poisson eqn is $4\pi G\rho = \nabla^2\Phi_g = \dfrac{d^2\Phi_g}{dz^2}$

Here, ρ is the total mass that generates Φ_g.

Integrate $\int_{-z}^{+z} dz$, using $\int_{-z}^{+z}\dfrac{d^2\Phi_g}{dz^2}\,dz = 2\,\dfrac{d\Phi_g}{dz}$ by symmetry.

So $$\boxed{2\pi G\Sigma = \frac{d\Phi_g}{dz} = -\frac{1}{n_*}\frac{d}{dz}\left(n_* c_z^2\right)}$$

Oort (1932) measured $n_*(z)$ and c_z [assumed constant] for bright F dwarfs and K giants.

This has been done several times since then. Result: $\boxed{\Sigma = 50\ M_\odot\ pc^{-2}}$

But observed mass is:

	$2\ M_\odot\ pc^{-2}$	H_2
	8 " "	HI
	35 " "	stars
	$45\ M_\odot\ pc^{-2}$	

So our disk does NOT contain much dark matter

Detecting Dark Matter: Gravitational Lensing

What is dark matter? There are 2 main suggestions:

WIMPS – Weakly Interacting Massive Particles – subatomic particles as yet unknown

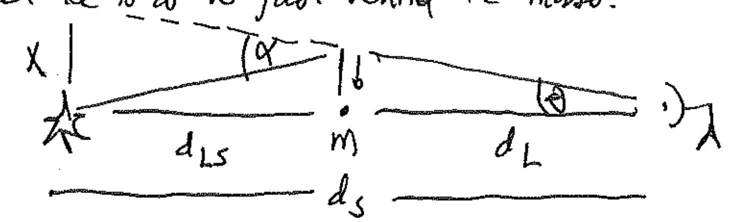
MACHOs – Massive Compact Halo Objects – dead stars, brown dwarfs, etc.

In the 1990's, an attempt was made to detect MACHOs through grav. lensing.
GR predicts that starlight is bent by any mass M, which acts as a lens, brightening the star.

For simplicity, let the star be just behind the mass.

What is θ?

$\alpha =$ bending angle $= \dfrac{4GM}{bc^2}$



$$x = \alpha\, d_{LS} \qquad \text{Also} \quad \theta = x/d_S$$

$$\text{so} \quad \theta = \alpha \frac{d_{LS}}{d_S} = \frac{4GM}{bc^2}\frac{d_{LS}}{d_S} \qquad \text{But } b = d_L \theta$$

$$= \frac{1}{\theta}\frac{4GM}{c^2}\frac{d_{LS}}{d_L d_S}$$

$$\text{Thus} \quad \theta_E \equiv \theta = \sqrt{\frac{4GM}{c^2}\frac{d_{LS}}{d_L d_S}} = 2\sqrt{\frac{R_S\, d_{LS}}{d_L\, d_S}}$$

"Einstein ring"

$R_S =$ Schwarzschild radius of lens

Suppose the star is in the LMC; $d_S = 16$ kpc. If the lens is in our halo, $d_L = 8$ kpc.
If $M = 0.5\, M_\odot$, $\theta_E = 5\times 10^{-4}$ arc sec.

As Macho passes in front of a star, the star will brighten when it enters the Einstein ring. If we know its proper motion, can find M from duration of the event.

From known halo mass, any background star has a probability $\sim 10^{-6}$ of being lensed. So a million stars in the LMC + our own MW bulge were monitored for several years. Some dozens of events were seen, but most were caused by MW bulge stars. Current thinking is that white dwarfs + dim stars account for no more than 10% of halo mass.