L21 - Cosmic Expansion

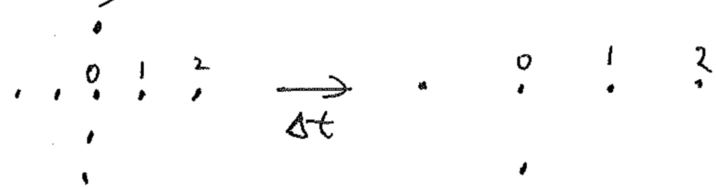
Prehistory

Slipher (1914) noticed that "spiral nebulae" (not yet known to be external galaxies] have redshifted spectra. First observed 12, then (1931) 40.

Hubble (1925) used Cepheids to find distances to 18 galaxies
(1929) - combined results with Slipher's redshifts to find
$$\boxed{V_r = H_0 d} \quad \text{Hubble's law}$$
units of H_0: km/s. Mpc (an inverse time)

de Sitter (1917) used GR (with $T_{\mu\nu}=0$) recall: $G_{\mu\nu} = \frac{-8\pi G}{c^4} T_{\mu\nu}$
to show that Universe can expand

Hubble Flow



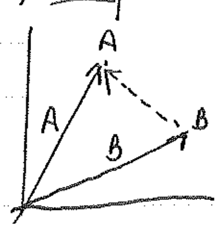
Consider a grid in which distance between any 2 points doubles in time Δt

> For points 0-1, distance goes from X_{01} to $2X_{01} \rightarrow V_{10} = X_{01}/t$

> For points 0-2, distance goes from X_{02} to $2X_{02} \rightarrow V_{20} = X_{02}/t$

But $X_{02} = 2X_{01} \longrightarrow \left.\begin{aligned} V_{20} &= 2X_{01}/t \\ V_{10} &= X_{01}/t \end{aligned}\right\}$ $\boxed{V \propto X}$

Moreover, every observer sees the same Hubble flow:



Consider 2 galaxies, located at $\vec{r}_A$ and $\vec{r}_B$.
Relative to origin
$$\vec{V}_A = H_0 \vec{r}_A$$
$$\vec{V}_B = H_0 \vec{r}_B$$

So velocity of A relative to B is
$$\vec{V}_A - \vec{V}_B = H_0 (\vec{r}_A - \vec{r}_B)$$
where $(\vec{r}_A - \vec{r}_B)$ is the distance of A relative to B ✓

$\boxed{V_r \text{ and } z}$

How did Hubble obtain V_r for his galaxies?
What he actually measured was the redshift $z \equiv \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0}$

He then interpreted the redshift via the Doppler formula.
Recall the longitudinal Doppler relation in special relativity

$$\lambda = \lambda_0 \sqrt{\frac{1+\beta}{1-\beta}} \qquad \beta \equiv V_r/c$$

Now $\frac{\lambda}{\lambda_0} = \frac{\lambda - \lambda_0}{\lambda_0} + \frac{\lambda_0}{\lambda_0} = z + 1 = \sqrt{\frac{1+\beta}{1-\beta}}$ What is $\beta(z)$?

$$(z+1)^2 = \frac{1+\beta}{1-\beta} \longrightarrow \quad (z+1)^2 - \beta(z+1)^2 = 1 + \beta$$
$$(z+1)^2 - 1 = \beta\left[(z+1)^2 + 1\right] \longrightarrow \boxed{\beta = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}} = \frac{V_r}{c}$$

Remember: Only z is measured.
V_r is an interpretation of this observation (and not strictly a correct one, since we assume Minkowski spacetime)

For $z \ll 1$, $(1+z)^2 \simeq 1 + 2z$

$$\beta \simeq \frac{1 + 2z - 1}{1 + 2z + 1} \approx \frac{2z}{2} = z \longrightarrow \boxed{\frac{V_r}{c} = z} \quad z \ll 1$$

Using Hubbles law, $V_r = H_0 d$ $\quad \boxed{d = \frac{V_r}{H_0} = \frac{c}{H_0}\frac{V_r}{c} = \frac{cz}{H_0}} \quad z \ll 1$

We will refine this latter formula using GR.

Value of H_0 and the Big Bang

It has been difficult to measure H_0 accurately, so it is conventional to write

$$\boxed{H_0 = 100\ h \ \ \mathrm{km\ s^{-1}\ Mpc^{-1}}}$$

The most accurate modern measurement comes, not from V_r of galaxies [see below], but from careful study of the cosmic microwave background (CMB).

This is radiation that was emitted billions of years ago, just now reaching us.
A satellite, the Wilkinson Microwave Anisotropy Probe (WMAP), looked at angular distribution of the CMB. Found $\boxed{h = 0.71}$ the current, preferred value.

When did the Big Bang occur?

Crudely, suppose the recession speed is constant in time:

Then $d = vt = H_0 dt \rightarrow$

$$\boxed{\begin{aligned} t &= \frac{1}{H_0} \\ &= 9.8\, h^{-1}\ \text{Gyr} \\ &= 13.8\ \text{Gyr}\ ;\ h = 0.71 \end{aligned}}$$

roughly:

(The true age is less, since the expansion H was bigger before)

Peculiar Velocities

- Consider M31, $D = 770$ kpc. It should be receding at $V_r = +55$ km/s for $h = 0.71$

Instead, it is approaching the Milky Way at $V_r = -119$ km/s!

This is due to the mutual attraction of the Milky Way and M31.

- Consider the Virgo cluster, $D = 16$ Mpc. It should be receding at $V_r = +1140$ km/s for $h = 0.71$

When V_r is measured accurately, it is less than this by $V_{pec} = 170$ km/s (peculiar velocity)

In a plot of V_r vs D, for clumps of galaxies, those which are close to Virgo all lie below the Hubble line.

Handout – Fig 7.8 of Sparke + Gallagher

The closer to Virgo, the farther V_r falls below the Hubble trend

In this case, Virgo is at the center of the Local Supercluster, while we are at the edge.

The gravity of the Supercluster pulls the MW toward Virgo, so that it recedes less quickly than in the Hubble flow.

If we correct V_r by the infall toward Virgo, we restore the Hubble flow [right panel of figure].

Peculiar velocities frustrate the measurement of H_0. Thus, most accurate measure of H_0 comes from the CMB

The Cosmological Principle

If we [carefully!] subtract off the peculiar velocity, $\boxed{\text{OR}}$ if we consider scales where the Universe is smooth ($D \gtrsim r_{max} \sim 30$ mpc), things simplify. The Universe is

- homogeneous (smooth)
- isotropic (expands equally fast in all directions)

That this is so is the basic underlying tenet of cosmology.

NB: The distance "30 mpc", & others like it, should really be "$30\,h^{-1}$ Mpc".

NB$_2$: The Hubble "constant" H_0 is the proportionality of V_r and d relatively nearby. or d and z
Moreover H is a function of time – $H(t)$.
It was larger in the past.

Redshift and Expansion

As the Universe expands, so does the λ of photons
Let $R(t)$ be the "size of the Universe" (to be defined)

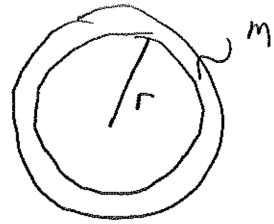
[We will prove this later using GR]

Then $\boxed{\lambda \propto R}$ Suppose photon has λ_{em} when emitted, $R = R_{em}$
" " λ_{obs} " observed, $R = R_{obs}$

$$\frac{\lambda_{obs}}{\lambda_{em}} = \frac{\lambda_{obs}}{\underbrace{\lambda_0}_{\text{lab value}}} = \boxed{1 + z = \frac{R_{obs}}{R_{em}}}$$

(Newtonian) Expansion of Dust Shell

Suppose the Universe is filled with pressureless "dust", whose only effect is gravitational



Consider an expanding shell, mass M, radius $r(t)$.

Energy conservation: $K(t) + U(t) = E = \text{constant}$ [during expansion, K drops, U rises]

Write E in terms of k and ϖ ("varpi")

"ϖ" labels the mass shell. We can make ϖ the present shell radius:

$$\varpi \equiv r(t_0)$$

"k" has units of $1/L^2$ — curvature constant

Write: $\boxed{E = -\tfrac{1}{2} m k c^2 \varpi^2}$

Energy equation becomes: $\frac{1}{2} m v^2 - \frac{G M_r m}{r} = -\frac{1}{2} m k c^2 \varpi^2$

But $M_r = \frac{4}{3}\pi r^3 \rho(t) = \text{const}$, so $\boxed{v^2 - \frac{8\pi}{3} G \rho r^2 = -k c^2 \varpi^2}$

if $k > 0$	$E < 0$	expansion will halt + reverse	closed Universe
$k < 0$	$E > 0$	expansion continues forever	open Universe
$k = 0$	$E = 0$	$v \to 0$ as $r \to \infty$	flat Universe

Finally, I rewrite $r(t)$ as

$$r(t) = R(t)\varpi$$

$R(t)$ is dimensionless, scale factor

NB: $R(t_0) = 1$

$r(t)$ = coordinate distance — increases w/ time

ϖ = comoving coordinate — follows shell

All shells, of differing ϖ, have same $R(t)$. And $\boxed{R = \frac{1}{1+z}}$ [$z = 2 \to R$ was $1/3$ of current size]

Since $\rho R^3 = \text{const} \to \rho \propto \frac{1}{R^3} = (1+z)^3$

$\boxed{\rho = \rho_0 (1+z)^3}$ where ρ_0 is the present density

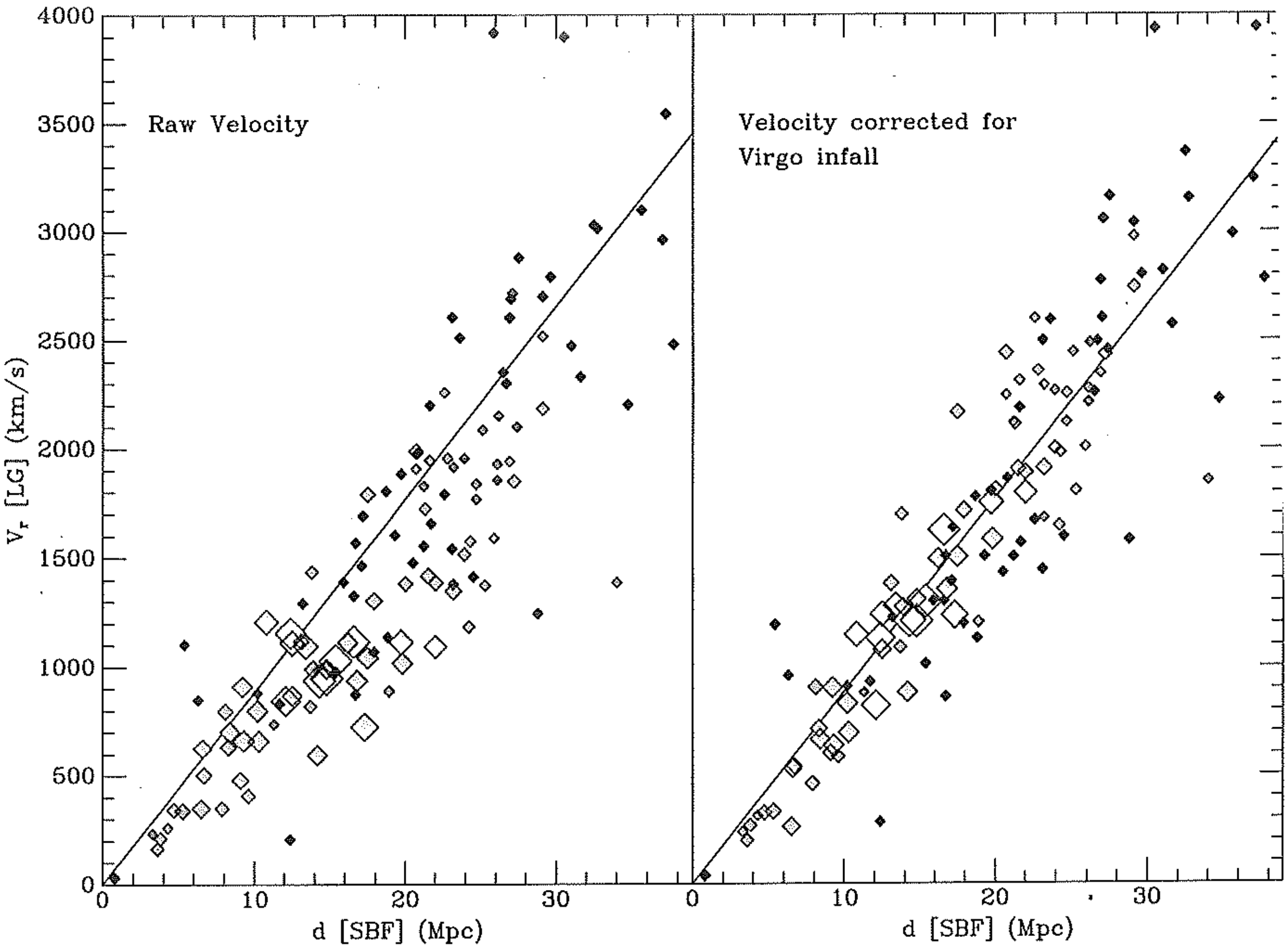


Figure 7.8 Diamonds show average recession speed V_r, measured relative to the Local Group, for groups of galaxies in Figure 7.2. The two largest white symbols are two clumps within the Virgo cluster; others decrease in size to show distance from Virgo. Left, velocity V_r falls further below the linear trend, the closer the group is to Virgo; right, after correction for Virgocentric infall – J. Tonry.