L25 - The Microwave Background: CMB

Origin of the Radiation

The early Universe was not only much denser, but also much hotter. From the 1st law of thermodynamics -

$$\Delta U = -P\Delta V \quad \text{for a parcel of given mass.}$$

So U (internal energy) decreases with time.

That is, U was higher in the past. Since temperature is (essentially) the internal energy per unit mass, T was higher in the past.

More precisely, the fluid equation reads:

$$\rho R^{3(1+w)} = \text{const}$$

Consider a Universe in which the main component is (blackbody) radiation. Then

$\rho c^2 =$ energy density $= aT^4$. Recall that w is defined by

$$P = w\rho c^2 = wu$$

For radiation, $P = \frac{1}{3}aT^4 = \frac{1}{3}u \longrightarrow w = \frac{1}{3}$. Since $\rho \propto T^4$,

$$T^4 R^{3(1+\frac{1}{3})} = T^4 R^4 = \text{const} \rightarrow \boxed{T \propto \frac{1}{R}}$$

Since $R=1$ today, we have $\boxed{RT = T_0}$

where T_0 is the present-day temperature of the radiation. What is it?

In 1948, Alpher & Gamow realized that most of the He in the Universe must have been made shortly after the Big Bang. This was the era of cosmological nucleosynthesis, which created (only) H, He, Li, and Be.

They estimated that, at the time of He synthesis, $R = 3\times10^{-9}$ and $T = 10^9$ K.

So they found $\boxed{T_0 \simeq 3\text{ K}}$

The modern, precise value is $\boxed{2.73\text{ K}}$

From Wien's law, $\boxed{\lambda_{max} = 1\text{ mm}}$, indeed in the microwave regime.

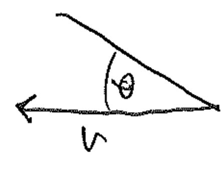
Anisotropy

The radiation intensity is very nearly the same in all directions. That is, T is almost isotropic. But there are small variations of T as a function of angle on the sky.

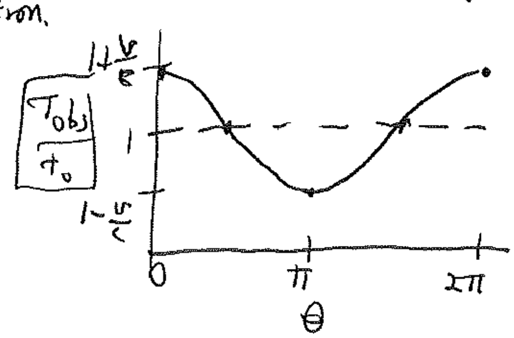
dipole anisotropy

The most important source of anisotropy is caused by the motion of the Earth relative to the Hubble flow. This "dipole anisotropy" is

$$T_{obs} = T_0 \left(1 + \frac{v}{c}\cos\theta\right)$$



Here, θ is the angle between the direction of observation and the direction of motion.



Observationally, $v = 371$ km/s

so $\frac{v}{c} \doteq \frac{4\times10^2}{3\times10^5} \approx 10^{-3}$

After this big anisotropy is subtracted off, the remaining anisotropy is of order 10^{-5}. Its detailed analysis yield the values of $\Omega_b, \Omega_m, \Omega_\Lambda$ etc used today.

The Two-Component Universe

We originally considered the Universe to consist only of pressureless dust. But we also need to account for the presence of the radiation which we now observe as the CMB.

In fact, $\rho_m \propto R^{-3}$, while $\rho_{rel} \propto R^{-4}$ [Since $\Lambda = $ const, one can ignore dark energy in the early Universe]. So the energy of radiation dominated the very early Universe.

The Friedmann equation was

$$\left[\frac{1}{R}\left(\frac{dR}{dt}\right)^2 - \frac{8\pi G}{3}\rho\right]R^2 = -kc^2$$

The equation now becomes:

$$\left[\frac{1}{R}\left(\frac{dR}{dt}\right)^2 - \frac{8\pi G}{3}\left(\rho_m + \rho_{rel}\right)\right]R^2 = -kc^2 \qquad [\text{neglecting } \rho_\Lambda]$$

ρ_m: The density of all matter, both baryons and dark matter (CDM)
ρ_{rel}: The equivalent mass density of relativistic particles (hot, photons)
At any epoch, $u_{rad} = aT^4 \rightarrow \rho_{rel} = \frac{aT^4}{c^2}$

As always, $\frac{1}{R}\frac{dR}{dt} = H(t)$ The Friedmann equation becomes

$$H^2\left[1 - (\Omega_m + \Omega_{rel})\right]R^2 = -kc^2$$

where $\Omega = \rho/\rho_c$ and $\rho_c \equiv \frac{3H^2}{8\pi G}$

In the early Universe, $k \simeq 0$ and $\boxed{\Omega_m + \Omega_{rel} = 1}$

What is Ω_{rel} today? $\Omega_{rel} = \frac{\rho_{rel}}{\rho_c} = \frac{8\pi G \rho_{rel}}{3H^2} = \frac{8\pi G aT^4}{3H^2c^2}$

Inserting $T = 2.73$ K, $H = 71$ km s^{-1} Mpc^{-1}, $\boxed{\Omega_{rel,0} = 8\times10^{-5}}$
This is tiny compared to $\Omega_{m,0} = 0.27$.

From Radiation Era to Matter Era

The variation of ρ_{rel} with scale factor is $\rho_{rel} = \rho_{rel,0}\, R^{-4}$

In contrast, ρ_m varies as $\rho_m = \rho_{m,0}\, R^{-3}$

Radiation dominated early on. The transition to matter-domination occurred when

$$\rho_m = \rho_{rel} \longrightarrow \rho_{rel,0}\, R^{-4} = \rho_{m,0}\, R^{-3}$$

Call the scale factor at that time $R_{r,m}$. Then

$$\boxed{R_{r,m} = \frac{\rho_{rel,0}}{\rho_{m,0}} = \frac{\Omega_{rel,0}}{\Omega_{m,0}} = \frac{8\times10^{-5}}{0.27} = 3\times10^{-4}}$$

Since $R = \frac{1}{1+z}$, $\quad z = \frac{1}{R} - 1 \rightarrow \boxed{z_{r,m} = 3{,}270}$

Recall that $T(\text{rad}) \propto R^{-1}$.

So T_{rad} at the time of this transition was $\boxed{T_{r,m} = \frac{T_0}{R_{r,m}} = 8{,}900\ K}$ optical

To find out when the transition occurred, we need —

Evolution of the Scale Factor

Substitute the R-dependence of ρ_m and ρ_{rel} into the new and improved Friedmann equation:

$$\left[\frac{1}{R}\left(\frac{dR}{dt}\right)^2 - \frac{8\pi G}{3}\left(\frac{\rho_{m,0}}{R^3} + \frac{\rho_{rel,0}}{R^4}\right)\right]R^2 = -kc^2$$

$$\left(\frac{dR}{dt}\right)^2 - \frac{8\pi G}{3}\left(\frac{\rho_{m,0}}{R} + \frac{\rho_{rel,0}}{R^2}\right) = -kc^2$$

Since, for $z \gg 1$, $\Omega_0 \simeq 1$, set $k=0$:

$$\frac{dR}{dt} = \sqrt{\frac{8\pi G}{3}}\left(\frac{\rho_{m,0}}{R} + \frac{\rho_{rel,0}}{R^2}\right)^{1/2}$$

$$R\frac{dR}{dt} = \sqrt{\frac{8\pi G}{3}}\left(\rho_{m,0} R + \rho_{rel,0}\right)^{1/2}$$

$$\boxed{\int_0^R \frac{R\,dR}{\sqrt{\rho_{m,0} R + \rho_{rel,0}}} = \sqrt{\frac{8\pi G}{3}}\,t}$$

Setting the upper limit equal to $R_{r,m}$, we find* $\boxed{t_{r,m} = 6 \times 10^4\ yr}$

For $t \ll t_{r,m}$ (radiation era) $\quad \int_0^R \frac{R\,dR}{\sqrt{\rho_{rel,0}}} = \sqrt{\frac{8\pi G}{3}}\,t \rightarrow t \propto R^2 \rightarrow \boxed{R \propto t^{1/2}}$

For $t \gg t_{r,m}$ (matter era) $\quad \int_0^R \frac{R\,dR}{\sqrt{\rho_{m,0} R}} = \sqrt{\frac{8\pi G}{3}}\,t \rightarrow t \propto R^{3/2} \rightarrow \boxed{R \propto t^{2/3}}$

Decoupling

In the very early Universe, photons scattered very frequently off free electrons. Thus, photons and electrons were locked to the same temperature (the radiation temperature).
Coulomb interactions between electrons and protons also kept the protons to this temperature.

Now the density of electrons fell w/ time. So eventually, this scattering became rare. The photons & matter then decoupled. The radiation evolved on its own and became today's CMB.

recombination However, the real cause of decoupling was not the decline in the electron density, but the ~~fact~~ fact that electrons and protons combine to form H atoms. When this "recombination" occurred, the opacity fell enormously. The CMB photons we see today originated from the "surface of last scattering" that surrounds us.

To find R at decoupling, we solve the Saha equation. This gives f, the fraction of ionized H atoms as a function of the neutral density ρ_b and the temperature T. Both ρ_b and T are known functions of R.

So, if we set $f = 1/2$ (say), we can obtain R at decoupling. We find:

$$R = 7 \times 10^{-4}, \text{ so that}$$

$$\boxed{z = \frac{1}{R} - 1 = 1100}$$

The radiation temperature at decoupling was $\boxed{T = 3000\ K}$.

The time was $\boxed{4 \times 10^5\ yr}$, significantly later than $t_{r,m}$.