L26: The Early Universe

Limits to Theory

At extremely early times, quantum effects and GR effects become comparable. This Planck time is

$$t_p = \sqrt{\frac{\hbar G}{c^5}} = 5 \times 10^{-44}\,s$$

There is a corresponding Planck length, given by

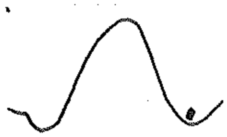
$$l_p = c t_p = \sqrt{\frac{\hbar G}{c^3}} = 2 \times 10^{-35}\,m$$

Finally, one may define the Planck mass m_p, by equating $\frac{G m_p}{c^2} = l_p$

$$m_p = \frac{l_p c^2}{G} = \sqrt{\frac{\hbar G}{c^3}}\,\frac{c^2}{G} = \sqrt{\frac{\hbar c}{G}} = 2 \times 10^{-8}\,kg$$

Theory cannot describe the Universe for $t < t_p$, when the horizon was smaller than l_p. Any primordial black holes could not have mass less than m_p, or else they would evaporate in a time less than t_p.

For $t < t_p$, it is thought that all 4 forces had the same magnitude. Forces separate out through the process of symmetry breaking:



Although the potential $\phi(x)$ is symmetric, the stable equilibrium states are not. The change of the system from the unstable to stable equilibrium point leads to new forces.

[This is really a plot of $u(\phi)$, where ϕ is the field value.]

At $t \sim t_p$, gravity separated from the other 3 ~~forces~~ forces.
At $t \sim 10^{-36}\,s$, the strong nuclear force separated from the electroweak force
At $t \sim 10^{-11}\,s$, the electromagnetic and weak forces separated

Quarks were created then and, at $10^{-5}\,s$, combined to give hadrons.

Hadrons composed of 3 quarks are baryons ($p + n$)
2 mesons

Problems in Big Bang Cosmology

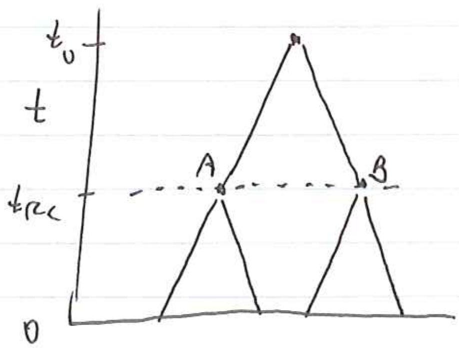
- Flatness Problem

For the simplest dust Universe,

$$\left(\frac{1}{\Omega_0}-1\right) = \left(\frac{1}{\Omega}-1\right)(1+z)$$

Any small deviation of Ω from 1 when $z>>1$ would have lead to Ω_0 very different from 1 now. Conversely, the early Ω must have been incredibly close to 1. Why?

- Horizon Problem



Suppose, today at t_0, we observe two points A + B in the CMB. These points were separated by a large angle at decoupling (t_{dec}). If the points are separated today by more than $\boxed{2^\circ}$, then their past light cones do not intersect at $t=0$. In other words, these regions were never in causal contact.* So why do they have nearly the same T?

Inflation

Suppose the main source of energy density immediately after the Big Bang was a quantum field that underwent spontaneous symmetry breaking.

($t<10^{-36}$ s)
Before this happened, the Universe was a false vacuum and $R(t)$ was much smaller than in standard cosmology. The growth of the particle horizon allowed vast regions to be in causal contact. This "solves" the horizon problem.

Upon symmetry breaking, $R(t)$ underwent huge growth - inflation. For $t>10^{-34}$ s, $R(t)$ evolved according to the standard model

Because of the fast rise in $R(t)$, Ω was forced to unity. Proof:

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* In other words, they are situated outside of each others' particle horizons.

$\Omega \equiv \frac{\rho(t)}{\rho_c(t)}$; where $\rho_c \equiv \frac{3H^2}{8\pi G} = \frac{3}{8\pi G}\frac{1}{R^2}\left(\frac{dR}{dt}\right)^2$

The Friedmann equation is $R^2H^2 - \frac{8\pi G\rho}{3}R^2 = -kc^2$.

Multiply through by $\frac{3}{8\pi G\rho_c}$: $\frac{R^2}{\rho_c}\frac{3H^2}{8\pi G} - \frac{\rho}{\rho_c}R^2 = -\frac{3kc^2}{8\pi G\rho_c}$

$$R^2(1-\Omega) = -kc^2\frac{R^2}{H^2} = -kc^2\frac{R^4}{\left(\frac{dR}{dt}\right)^2}$$

So that $\boxed{\Omega = 1 + \frac{kc^2}{(dR/dt)^2}}$ Thus, a large dR/dt indeed drives Ω close to 1.

In more detail, ρ for the false vacuum was constant in time, so $R(t)$ increased exponentially, as during the Λ era.

Growth of Fluctuations

Prior to recombination, the Universe consisted of a "photon-baryon fluid." When inflation occurred, the size of perturbed regions was stretched out <u>much larger</u> than the horizon scale. The perturbations were then frozen into the Hubble flow & were not causally connected.

How did $\delta\rho/\rho$ increase with time?
Each perturbed volume acted like a little closed Universe, with $k>0$, but expanding w/ the bkgrnd flow.

According to Friedmann, the background ρ obeyed: $H^2R^2 - \frac{8\pi G}{3}\rho R^2 = 0$
the interior ρ' obeyed: $H^2R^2 - \frac{8\pi G}{3}\rho' R^2 = -kc^2$

Subtracting: $\frac{8\pi G}{3}(\rho'-\rho)R^2 = kc^2$

$$\boxed{\frac{\delta\rho}{\rho} = \frac{\rho'-\rho}{\rho} = \frac{3kc^2}{8\pi G\rho R^2}}$$

> During the radiation era, $\underbrace{\rho \propto R^{-4},\ R \propto t^{1/2}}_{\rho \propto t^{-2}} \to \rho R^2 \propto t^{-2}t = t^{-1}$

Thus $\boxed{\delta\rho/\rho \propto t^1}$ slow, linear growth – because of expansion

> During the matter era, $\underbrace{\rho \propto R^{-3},\ R \propto t^{2/3}}_{\rho \propto t^{-2}\ \text{(again)}} \rightarrow \rho R^2 \propto t^{-2} t^{4/3} = t^{-2/3}$

Thus $\boxed{\delta\rho/\rho \propto t^{2/3}}$ <u>sub-linear</u> growth

<u>Fall of the Jeans Mass</u>

Recall the Jeans length

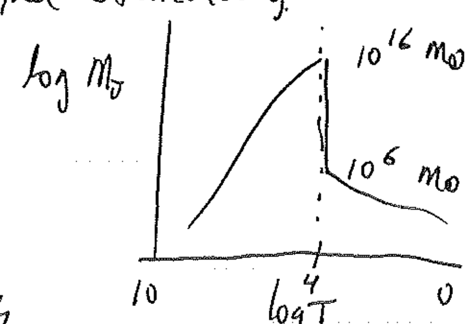
$$\lambda_J \doteq c_s t_{ff} \simeq \frac{c_s}{\sqrt{G\rho}} \longrightarrow M_J = \rho \lambda_J^3 = \frac{c_s^3}{G^{3/2}\rho^{1/2}}$$

> <u>prior to recombination</u>, c_s was very high (in the photon-baryon fluid)

$$c_s^2 = \frac{dP}{d\rho} \qquad P = \frac{1}{3} u_{rel} = \frac{1}{3}\rho_{rel} c^2 \longrightarrow \frac{dP}{d\rho} = \frac{1}{3}c^2$$

$$\longrightarrow c_s = \frac{1}{\sqrt{3}} c\,!$$

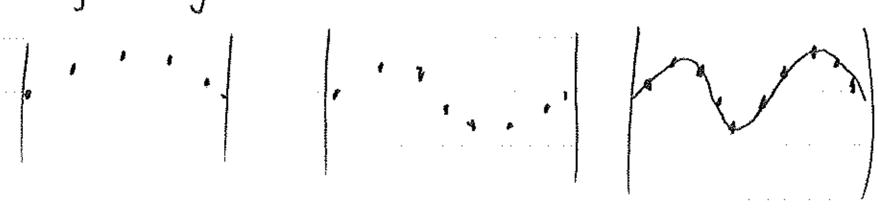
> <u>After recombination</u>, c_s (& therefore M_J) dropped dramatically.



<u>Baryon Acoustic Oscillations</u>

Not long before recombination, the horizon size grew to engulf the perturbations. ($t \sim 10^5$ yr). At this time, the matter inside the perturbation was causally connected. However, $M \ll M_J$, since the latter was so huge.

Thus, the perturbations did not collapse, but <u>oscillated</u>. In fact, they oscillated in a superposition of many normal modes.



This complex pattern of oscillations persisted until recombination.

After that point, photons broke free of the gas. The anisotropy of the CMB radiation thus carries the imprint of the acoustic oscillations.

It is through analysis of the power spectrum* of the CMB, that WMAP obtained the current values for $\Omega_0, \Omega_m, \Omega_b, \Omega_\Lambda$.

The Problem of Slow Growth

The relative density perturbation $\delta\rho/\rho$ increases so slowly with time that it needed to have been [relatively] large even at recombination.

For example, a $z=6$ quasar $\left(a = \frac{1}{1+6} = 0.14\right)$, formed at $t_q = 9\times10^8$ yr. What was $\delta\rho/\rho$ at $t = t_{rec}$?

It must have been large enough so that $\delta\rho/\rho$ reached unity at $t = t_q$.

$$1 = \left(\frac{\delta\rho}{\rho}\right)_{trec}\left(\frac{t_q}{t_{rec}}\right)^{2/3}$$

Here, I am assuming that most of the growth occurred during the matter era. Using $t_{rec} = 4\times10^5$ yr,

$$\left(\frac{\delta\rho}{\rho}\right)_{trec} = \left(\frac{9\times10^8}{4\times10^5}\right)^{-2/3} = 5\times10^{-3}$$

The problem is that the observed CMB fluctuations, after subtracting the dipole anisotropy, are of order 10^{-5}!

Proposed solution: It is the dark matter that had perturbations of order 10^{-3} at the time of decoupling. Baryons fell into the dark matter halos and subsequently developed much larger relative perturbations.

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* Unlike the power spectrum for the galaxy correlation function, the independent variable is not the wave number k, but [essentially] the angular separation θ.