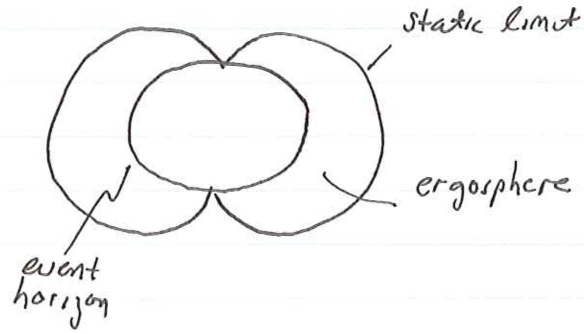
L6: Black Holes II

Charged and Spinning BH's

no hair

One can generalize from Schwarzschild to "Kerr–Newman" metric, representing a BH with m, Q, and $a \equiv L/M$. There is a "no hair" theorem: a BH can have no other characteristics.

Assuming $Q = 0$, there is a maximum $L = \frac{GM^2}{c}$. A spinning BH looks like:



Inside the "static limit" no particle can be stationary. [The world line of a stationary observer becomes space like.] This is an example of "frame dragging." All particles must rotate in the same direction as the BH.*

BH Thermodynamics

First law: Energy conservation — the mass/energy flowing into the horizon increases the mass of the BH.

Second law: The surface area of a BH, $A = 4\pi \left(\frac{2GM}{c^2}\right)^2$, can never decrease in any process.

This is reminiscent of entropy in the usual 2nd law. In fact, S is defined as

$$S \equiv \frac{k_B c^3}{4G\hbar} A$$

The temperature is $\qquad T = \frac{\hbar c^3}{8\pi G k_B M} = 10^{-7} K \left(\frac{M_\odot}{M}\right)$

↓

Rough estimate λ_T is the "thermal wavelength" (roughly mean distance between photons in a BB). Use $k_B T = h\nu = hc/\lambda_T$ to find λ_T. Equate to $R_s = 2GM/c^2$.

Hawking radiation

Hawking (1974) showed that a BH radiates (photons, particles) as if it had this T.

Gravity Probe B

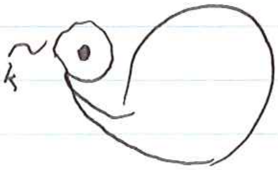
* Another example is Gravity Probe B: the Stanford experiment by Francis Everitt. Measured precession of 4 gyroscopes on board a satellite. The spin of the gyroscope is "parallel transported", & therefore precesses in the lab frame.

Finding BH's: Categories

- Stellar mass ($\sim 10\ M_\odot$) — binary companion — seen ✓
- intermediate mass ($10^3 - 10^5\ M_\odot$) — could be seen by tidal disruption of stars — ~~not yet~~ seen a few
- super-massive ($10^6 - 10^{10}\ M_\odot$) — centers of galaxies w/ bulges — seen ✓

Stellar-Mass BHs

disk



These are all <u>X-ray variables</u>

> <u>Cygnus X-1</u>

One of the strongest X-ray sources. Suspected of being a BH since the 1970's

One companion is a [visible] blue supergiant star. It orbits an invisible companion, with a separation of 0.05 AU

The supergiant is distorted tidally, & the resulting fluctuations in L ($P = 5.6$ days) help give the <u>inclination</u> of the orbit

accurate parallax obtained

Orosz et al 2011, ApJ, <u>742</u>, 84 Did careful modeling of both the V_r of visible star (from spectral lines) and photometry. They had first found $D = 1.86$ kpc from parallax with VLBA (~~mas~~ arcsec).

Result: $M_{visible} = 19\ M_\odot$ $M_{BH} = 15\ M_\odot > 3\ M_\odot$ (M_{max}, neutron star)

BH spin

In a subsequent paper, they determined the inner edge of the accretion disk, using a disk model fit to the X-ray observations. Assuming this edge represents the last stable orbit, found $R_{inner} = 1\ GM_{BH}/c^2 \rightarrow$ the spin is almost maximal.

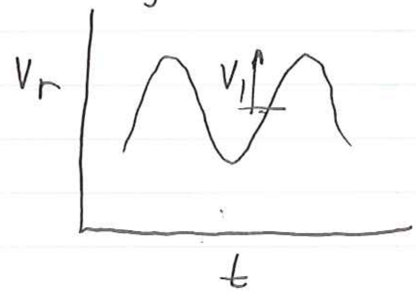
> IC 10 X-1 (Silverman & Filippenko 2008)

IC 10 is a Local Group galaxy – an "irregular starburst", containing WR stars.

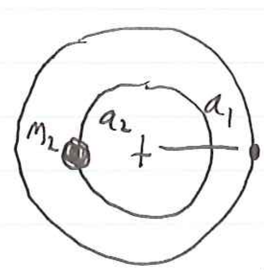
most massive to date

IC 10 X-1 is a periodically variable X-ray source. A WR's spectrum has the same period ($P = 36$ hours), so must be part of binary.

In fact, the $V_r(t)$ curve is sinusoidal, meaning that $e \simeq 0$.



Let V_1 be the amplitude of the V_r-curve.
Let m_1 be the mass of the WR star, and m_2 the mass of the invisible star.
→ Then we can almost find m_2, knowing V_1 and P:



$m_2 a_2 = m_1 a_1$

$a_{tot} \equiv a_1 + a_2$ (star separation)

Assuming circular orbits, consider $F = ma$:

For m_2: $\dfrac{G m_1 \cancel{m_2}}{a_{tot}^2} = \cancel{m_2}\,\omega^2 a_2$

For m_1: $\dfrac{G \cancel{m_1} m_2}{a_{tot}^2} = \cancel{m_1}\,\omega^2 a_1$

adding: $\boxed{\dfrac{G M_{tot}}{a_{tot}^3} = \omega^2 = \left(\dfrac{2\pi}{P}\right)^2}$ Generalized Kepler

Now $V_1 = \dfrac{2\pi a_1}{P} \rightarrow V_1^3 P^3 = (2\pi)^3 a_1^3$

$P^2 = \dfrac{(2\pi)^2 a_{tot}^3}{G M_{tot}}$

$\dfrac{V_1^3 P}{2\pi G} = \left(\dfrac{a_1}{a_{tot}}\right)^3 M_{tot}$

$a_{tot} = a_1 + a_2 = a_1\left(1 + \dfrac{a_2}{a_1}\right) = a_1\left(1 + \dfrac{m_1}{m_2}\right) \rightarrow \dfrac{a_1}{a_{tot}} = \left(1 + \dfrac{m_1}{m_2}\right)^{-1} = \dfrac{m_2}{M_{tot}}$

So $\boxed{\dfrac{V_1^3 P}{2\pi G} = \dfrac{m_2^3}{M_{tot}^3} M_{tot} = \dfrac{m_2^3}{M_{tot}^2}}$

observed desired

[more generally, $\dfrac{V_1^3 P}{2\pi G}(1-e^2)^{3/2} = \dfrac{m_2^3 \sin^3 i}{M_{tot}^2}$ mass function
Here, $e = 0$, $i = \pi/2$]

From observations, $\dfrac{V_1^3 P}{2\pi G} = 7.64\ M_\odot \equiv f_0$

Thus, $\boxed{m_2 = f_0\left(1 + \dfrac{m_1}{m_2}\right)^2}$

From stellar evolution, m_1 lies between 17 and 35 $M_\odot$.

We have, then,	M_1 (WR)	M_2 (BH)	
	17	23	all $\gg 3\ M_\odot$
Currently, this is the most	25	28	
massive stellar-type BH	35	33	

The Galactic Center: Phenomenology

rising n_* of young stars

The GC is at the center of the Galactic bulge ($r \sim 2$ kpc), which consists largely of low-mass, old stars. However, stars within the central 100 pc have $n_* \sim r^{-2}$ and appear to be forming continuously.

Sgr B2

Star formation is fueled by gas in the "central molecular zone", which contains (among other clouds) Sgr B2, the most massive GMC in the Galaxy.

There are several dense and massive clusters near the GC - the Arches, the Quintuplet, and the Central Cluster. Each has 100's of O stars! In fact they have 10% of all O stars in the Galaxy.

Within the Central Cluster is a ~~young~~ strong radio source - Sgr A* - the BH!

There are 2 separate populations of young stars surrounding Sgr A* -

disk stars

(1) 0.04 pc $< r < 0.4$ pc — Massive young stars, giants (eg WRs) orbiting in two mutually inclined disks

S stars

(2) $r < 0.04$ pc — Main-sequence B stars $3-15\ M_\odot$, $t_{MS} \sim 4\times10^8$ yr seem to be orbiting isotropically, not in a disk. Called "S stars."

IR movie

These are the test particles from which M_{BH} is derived. [show Ghez movie]

$$\boxed{M_{BH} = 3\times10^6\ M_\odot}$$

Alternative to BH?

Maoz (1998) - In a super-dense, dark cluster (eg white dwarfs), members would merge, creating a BH anyway.

Dynamical Influence of BH

$R_S = \frac{2GM_{BH}}{c^2} = 10^{12}$ cm ($\theta = 9$ μ arc sec) TINY

Tidal radius [where stars are torn apart] $R_t \sim R_* \left(\frac{M_{BH}}{M_*}\right)^{1/3} = 10^{13}$ cm
90 μ″ TINY

$F_{tidal} \sim \boxed{\frac{GM_{BH} R_*}{R_t^3}} \sim F_{self-grav} \sim \boxed{\frac{GM_*}{R_*^2}}$

Radius of Influence BH dominates motions out to where $\Sigma M_* \sim M_{BH}$
$r \sim 10^{19}$ cm (3 pc)

The S stars are not in danger of plunging into the BH or being torn apart, but they are deep in the potential well of the BH.

Problem: The Youth Paradox

The S stars (+ others in the disk) are much younger than the GC itself (10^{10} yr). If they formed in situ, the parent cloud must have been farther out than R_t:

Roche density for cloud

$$r > R_t = R_{cl}\left(\frac{M_{BH}}{M_{cl}}\right)^{1/3} \longrightarrow \frac{M_{cl}}{R_{cl}^3} \sim \rho_{cl} > \frac{M_{BH}}{r^3} \quad \text{"Roche density"}$$

For $M_{BH} = 3\times10^6\, M_\odot$, $n_{cl} > 10^8 \left(\frac{r}{1\,pc}\right)^{-3}$ cm^{-3} much too high!

Problem: Origin of the S Stars

Assuming that young stars formed farther out and drifted in through the disk, why is their motion now isotropic?

Hills (1988) showed, through N-body simulations, that a binary approaching a BH is disrupted. One member can be shot out at high speed, V_{ej}, while the other plunges into a tight orbit around the BH.* over

$V_{ej} \sim (V_{orb} V_{break})^{1/2} \sim 1700$ km/s

Just recently, since 2006, about a dozen hypervelocity stars have been observed. These are B stars with $V \sim 1000$ km/s racing away from the GC. QED.

Uniformly Accelerated Particle – Alternate Derivation

Consider the particle's 4-velocity $\underset{\sim}{u}$. We know that $\boxed{\underset{\sim}{u}\cdot\underset{\sim}{u} \equiv \eta_{\mu\nu}u^\mu u^\nu = +c^2}$

Let $\underset{\sim}{a}$ be the 4-acceleration – $\underset{\sim}{a} \equiv d\underset{\sim}{u}/d\tau$

Then $0 = \frac{d}{d\tau}\left(\frac{c^2}{2}\right) = \frac{d}{d\tau}\left(\frac{1}{2}\underset{\sim}{u}\cdot\underset{\sim}{u}\right) = \underset{\sim}{u}\cdot\frac{d\underset{\sim}{u}}{d\tau} = \boxed{\underset{\sim}{u}\cdot\underset{\sim}{a} = 0}$

In the particle's instantaneous rest frame, $\underset{\sim}{u} = (c,0,0,0)$. But in that frame,
$\underset{\sim}{u}\cdot\underset{\sim}{a} = u^0a^0 - u^1a^1 - u^2a^2 - u^3a^3 = 0 \rightarrow a^0 = 0.$
Also in that frame, $d\tau = dt \rightarrow a^1 \equiv \frac{du^1}{d\tau} = \frac{d}{dt}\left(\frac{dx}{d\tau}\right) = \frac{d^2x}{dt^2} = g$

Therefore, the invariant $\boxed{\underset{\sim}{a}\cdot\underset{\sim}{a} = -g^2}$ ($\underset{\sim}{a}$ is spacelike)

Now consider the global Lorentz frame.

$\underset{\sim}{u}\cdot\underset{\sim}{u} = +c^2 \longrightarrow (u^0)^2 - (u^1)^2 = c^2 \quad (1)$

$\underset{\sim}{u}\cdot\underset{\sim}{a} = 0 \longrightarrow u^0a^0 - u^1a^1 = 0 \quad (2)$

$\underset{\sim}{a}\cdot\underset{\sim}{a} = -g^2 \longrightarrow (a^0)^2 - (a^1)^2 = -g^2 \quad (3)$

$g^2 \times (1)$: $(gu^0)^2 - (gu^1)^2 = c^2g^2 = (ca^1)^2 - (ca^0)^2$, using (3)

$g \times (2)$: $(gu^0)a^0 = (gu^1)a^1 \rightarrow gu^1 = (gu^0)\frac{a^0}{a^1}$

Thus $(gu^0)^2 - \left(\frac{a^0}{a^1}\right)^2(gu^0)^2 = (ca^1)^2\left[1 - \left(\frac{a^0}{a^1}\right)^2\right] \rightarrow gu^0 = ca^1$
$gu^1 = ca^0$

But $u^0 = \frac{dt}{d\tau} = \frac{c}{g}\frac{du^1}{d\tau}$

Since $u^1 = \frac{dx}{d\tau}$, $u^0 = \frac{c}{g}\frac{d^2x}{d\tau^2} = \frac{c\,dt}{d\tau}$ Integrating,

$t = \frac{1}{g}\frac{dx}{d\tau} + \text{const.}$ Now $u^1 = \frac{dx}{d\tau} = \frac{c}{g}a^0 = \frac{c}{g}\frac{du^0}{d\tau} = \frac{c^2}{g}\frac{d^2t}{d\tau^2}$

$t = \frac{c^2}{g^2}\frac{d^2t}{d\tau^2} + \text{const}$ Solution: $\boxed{t = \frac{c}{g}\sinh\left(\frac{g\tau}{c}\right)}$ The constant must be zero so that $t(\tau=0)=0$

Since, moreover, $\frac{dx}{d\tau} = \frac{c^2}{g}\frac{d^2t}{d\tau^2} = c\sinh(g\tau/c)$

we have: $\boxed{x = \frac{c^2}{g}\cosh\left(\frac{g\tau}{c}\right)}$ as before.