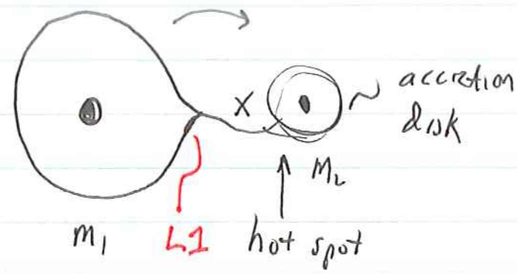
L7: Binaries and Accretion Disks

Stellar mass accretion disks are thought to arise in the following way:

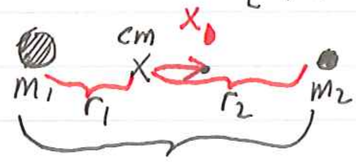


Two stars are orbiting in a binary.
One star evolves more quickly than the other, & eventually becomes a giant. Meanwhile, its companion is a compact object.

The giant swells, until gas spills over to the other star. This gas does not cross the cm but ~~strays~~ veers to one side (Coriolis force). The stream hits the edge of the accretion disk at a "hot spot." This radiates.

The gas spills through at the "inner Lagrange point," designated L1. This is the point where the total force – from m_1 from m_2, & from the centrifugal force [in the rotating frame] cancel.

$$[m_1 r_1 = m_2 r_2]$$



$$0 = \frac{-Gm_1}{(r_1+x)^2} + \frac{Gm_2}{(r_2-x)^2} + \omega^2 x$$

Use Kepler

$$0 = \frac{-Gm_1}{(r_1+x)^2} + \frac{Gm_2}{(r_2-x)^2} + \frac{G(m_1+m_2)x}{a^3}, \quad \text{where } a \equiv r_1 + r_2$$

$$0 = \frac{-(m_1/m_2)}{\left[\frac{r_1}{a} + \frac{x}{a}\right]^2} + \frac{1}{\left[\frac{r_2}{a} - \frac{x}{a}\right]^2} + (1 + m_1/m_2)(x/a)$$

But $\quad r_1/a = \dfrac{r_1}{r_1+r_2} = \dfrac{1}{1+r_2/r_1} = \dfrac{1}{1+m_1/m_2} \quad \left(= \dfrac{m_2}{m_{tot}}\right)$

$\quad r_2/a = \dfrac{r_2}{r_1+r_2} = \dfrac{r_2/r_1}{1+r_2/r_1} = \dfrac{m_1/m_2}{1+m_1/m_2}$

$q \equiv m_1/m_2$
$z \equiv x/a$

$$0 = \frac{-q}{\left[\frac{1}{1+q} + z\right]^2} + \frac{1}{\left[\frac{q}{1+q} - z\right]^2} + (1+q)z$$

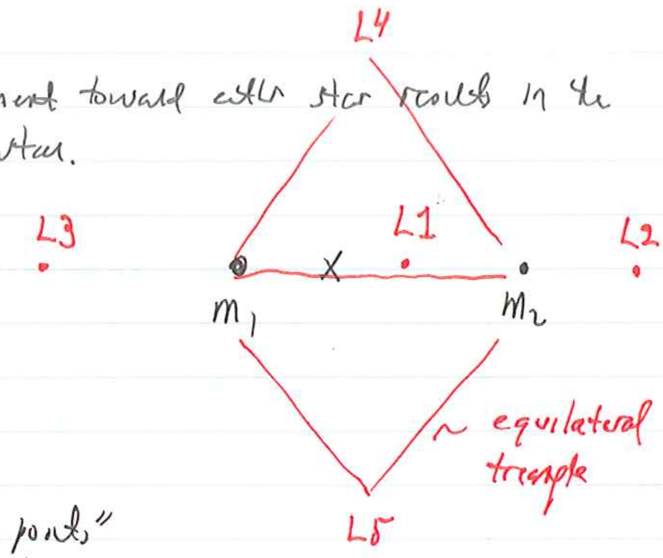
$$\boxed{0 = \frac{-q}{[1+(1+q)z]^2} + \frac{1}{[q-(1+q)z]^2} + (1+q)^3 z}$$

Gives z as a function of q.

(z increases with q.)

The L1 point is <u>unstable</u>. A displacement toward either star results in the test particle speeding up toward that star.

There exist two more places where the forces balance — along this line. These points, L2 and L3, are <u>also</u> unstable.



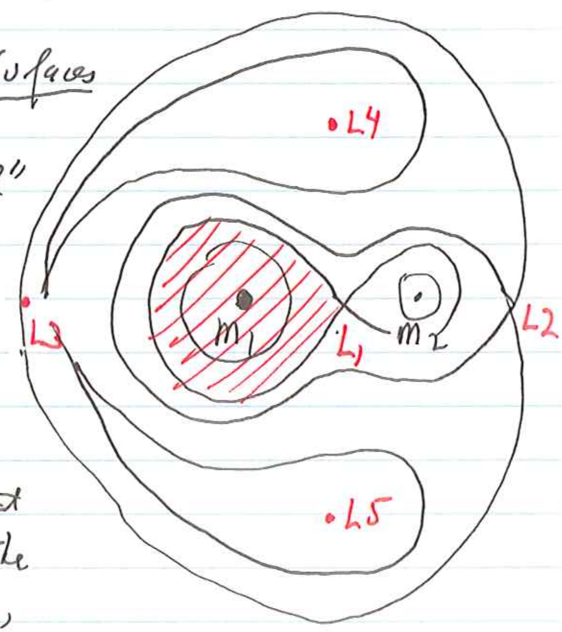
However, there are two "other Lagrange points" L4 & L5. Both are <u>stable</u>. In the solar system, the <u>Trojan asteroids</u> sit at the apex of an equilateral Δ with the Sun & Jupiter.

<u>Equipotential Surfaces</u>

Here the "potential" is the gravitational one from each star <u>plus</u> the "centrifugal potential"

$$U_{cen} = -\frac{1}{2}\omega^2 r^2$$

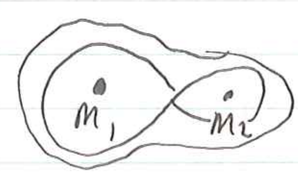
$$f = \left(\frac{F}{m}\right) = \frac{-dU_{cen}}{dr} = +\omega^2 r \checkmark$$



The volume enclosing the equipotential surface that ends in L1 is called the "Roche lobe". The gas in a star expands to fill its Roche lobe, then spills through L1.

<u>Binary Classification</u>

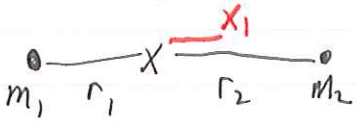
- detached — Neither star fills its Roche lobe
- semi-detached — Only one star does.



- contact — Both stars do. If they fill L2 also, there is a <u>common envelope</u>.

Co-Evolution of Binaries

This can be complex, but the general picture follows from ang. mom. conservation.



$$L^2 = \omega^2 (m_1 r_1^2 + m_2 r_2^2)^2$$
$$= \frac{G M_{tot}}{a^3} (m_1 r_1^2 + m_2 r_2^2)^2$$
$$= \frac{G M_{tot}}{a^3} m_1^2 r_1^4 \left(1 + \frac{m_2}{m_1}\frac{r_2^2}{r_1^2}\right)^2 = \frac{G M_{tot}}{a^3} m_1^2 r_1^4 \left(1 + \frac{r_2}{r_1}\right)^2 = \frac{G M_{tot}}{a^3} m_1^2 r_1^2 a^2$$

using $\frac{r_1}{a} = \frac{m_2}{M_{tot}}$

$$= (G M_{tot} a) \, m_1^2 \frac{r_1^2}{a^2} = (G M_{tot} a) \, m_1^2 \frac{m_2^2}{M_{tot}^2} = (G M_{tot} a) \mu^2$$

where $\mu \equiv \frac{m_1 m_2}{M_{tot}}$

So $L = \mu \sqrt{G M_{tot} a}$

Since L and M_{tot} are fixed $\rightarrow$ $0 = \frac{1}{\mu}\frac{d\mu}{dt} + \frac{1}{2a}\frac{da}{dt}$

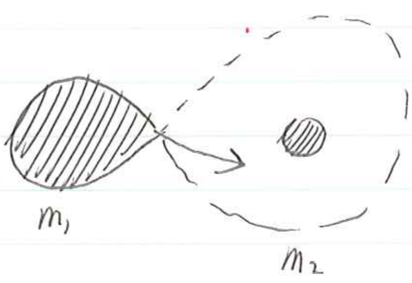
$$\dot{\mu} = \frac{\dot{m}_1 m_2 + m_1 \dot{m}_2}{M_{tot}} \rightarrow \frac{1}{\mu}\dot{\mu} = \frac{\dot{m}_1 m_2 + m_1 \dot{m}_2}{m_1 m_2 \cdot \cancel{M_{tot}}} \cdot \cancel{M_{tot}} = \frac{\dot{m}_1 (m_2 - m_1)}{m_1 m_2}$$

Thus, $\boxed{\frac{1}{a}\frac{da}{dt} = \frac{2\dot{m}_1 (m_1 - m_2)}{m_1 m_2}}$ And m_1 is bigger and is losing mass, $\dot{a} < 0$. Orbit shrinks. As m_1/m_2 decreases, X_1/a also decreases. L1 moves inward, and mass xfer accelerates.

The Algol Paradox

In a semi-detached binary, if the detached component is a normal star, we have an "Algol-like" system.

In the prototype, Algol, m_1 is a subgiant of spectral type K0, m_2 is a m-s star of type B8.



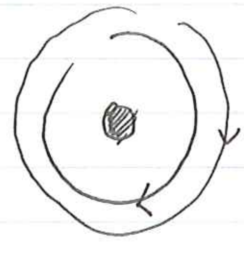
The system is eclipsing, & at the primary eclipse, the total brightness drops by a factor of two. This is only possible because m_1 is bigger in size, yet less luminous. This is only possible if m_2 has a lot more mass. And, in fact, $m_1 = 0.8\,M_\odot$ and $m_2 = 3.7\,M_\odot$.

PARADOX — The less massive star has evolved further than the more massive one!

RESOLUTION — m_1 used to be more massive. It filled its Roche lobe and quickly transferred mass to m_2. Once m_1 dropped below m_2, the transfer slowed. So all Algol's are in the situation — less massive transferring gas to more massive.

Accretion Disks: Basics

Consider again a semi-detached binary. After passing through the hot spot, gas spirals onto the detached star.
Each gas element is orbiting. To spiral inward, it <u>must lose angular momentum</u>.



The leading idea is that it transfers L <u>outward</u> through "viscosity." The inner ring shown has higher ω than the outer one. So the inner one is rubbed by the outer one. In effect, it donates L to the outer ring.

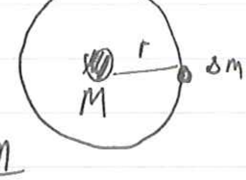
The essence of accretion disks is that <u>L flows outward & M flows inward</u>.

The nature of the viscosity is still not known. If the disks are (somehow) turbulent, the turbulence might provide a source. Or tangled B-fields – the "magnetorotational instability."

Temperature Profile

disks are (vertically) "optically thick"

Remarkably, one can derive $T(r)$ without knowing the viscosity! The main assumption is that the disk radiates like a blackbody at each radius.



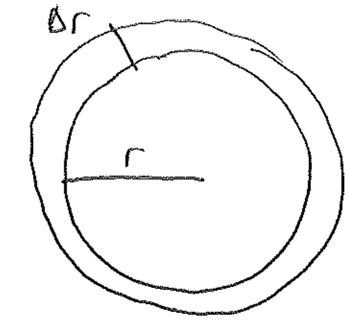
To a good approximation, each element Δm is on a keplerian, circular orbit.

$$U = -\frac{GM\Delta m}{r} \qquad K = \frac{1}{2}\Delta m v^2 \quad \text{but} \quad \frac{v^2}{r} = \frac{GM}{r^2}$$

$$= \frac{1}{2}\frac{GM\Delta m}{r} \longrightarrow \boxed{E_{tot} = -\frac{1}{2}\frac{GM\Delta m}{r}}$$

Total energy is negative (Δm is bound), and $\frac{1}{2}U$ in magnitude.
As Δm spirals inward, E_{tot} becomes <u>more</u> negative. The lost energy is radiated away by the <u>surface</u> of the disk.

In steady state, $\dot{M}$ (mass/time crossing a ring) is <u>constant</u>.



Through any ring, $\dot{m}\,\Delta t$ enters and
$\dot{m}\,\Delta t$ leaves.
So we may identify $\Delta m = \dot{m}\,\Delta t$
(mass of fluid element)

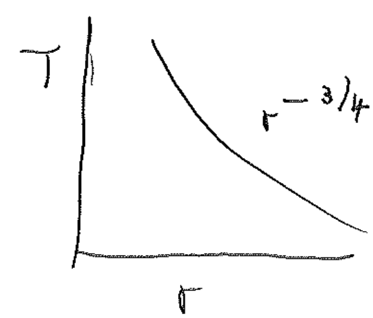
Question: How much energy does that element emit?
Call this quantity $\Delta^2 E$. It is small because (1) Δm is small, and (2) we only want energy emitted over a brief time Δt.

$$\Delta^2 E = \frac{dE_{tot}}{dr}\Delta r = +\frac{1}{2}\, G\frac{M\Delta m}{r^2}\,\Delta r = \frac{1}{2}\, \frac{GM\dot{m}\Delta t}{r^2}\,\Delta r$$

But $\Delta^2 E = \Delta L\,\Delta t \longrightarrow \Delta L = \frac{1}{2}\,\frac{GM\dot{m}}{r^2}\,\Delta r$

Since each ring radiates like a black body $\Delta L = \underbrace{2\pi r\,\Delta r}_{\Delta A}\,\sigma T^4 \times 2$ (two faces of disk)

$$\sigma T^4 = \frac{GM\dot{m}}{8\pi r^3} \longrightarrow \boxed{T = \left(\frac{GM\dot{m}}{8\pi\sigma r^3}\right)^{1/4}}$$



Total Luminosity

$$dL = \frac{1}{2}\,\frac{GM\dot{m}\,dr}{r^2}$$

$$L_{tot} = \frac{1}{2}\,GM\dot{m}\int_R^\infty \frac{dr}{r^2}$$

where R is the stellar radius, or inner edge of the disk

$$\boxed{L_{tot} = \frac{1}{2}\,\frac{GM\dot{m}}{R}}$$

If each fluid element released all of its energy, we would have $L_{acc} = \frac{GM\dot{m}}{R}$
So half of the energy released must be deposited on the rotating star.
Some of it could spin up the star.
Similarly, what happens to the angular momentum? As it flows outward the outer edge of the disk must expand.