L9 – Distances, Magnitudes, + Dust

Apparent Magnitude

The Greeks first assigned "apparent mag = 1" to the brightest stars, those highest in rank. The lowest in rank were $m=6$. So a numerically higher m-value corresponds to a dimmer star. An $m=6$ star was about 1% as bright as an $m=1$ star.

In modern times, this was made precise. A difference of 5 mag = a factor of 100 in received flux. Each Δm corresponds to a fixed $\Delta \log F$. A difference of 1 mag is a factor of $100^{1/5}$ in flux.

$$m = A \log F + B \qquad \text{where } A + B \text{ are constants}$$

We know that $\Delta m = -5 \longrightarrow \Delta \log F = +2$

$$-5 = 2A \Longrightarrow A = -2.5$$

$$\boxed{m = -2.5 \log F + B}$$

Given two stars of m_1 and m_2, what is F_2/F_1?

$$m_2 - m_1 = -2.5 \log\left(\frac{F_2}{F_1}\right)$$

$$-\frac{2}{5}(m_2 - m_1) = \frac{+2}{5}(m_1 - m_2) = \log\left(\frac{F_2}{F_1}\right)$$

$$\boxed{\frac{F_2}{F_1}} = \boxed{10^{\frac{2}{5}(m_1 - m_2)}} = \boxed{100^{\frac{m_1 - m_2}{5}}}$$

Absolute Magnitude

This is a measure of intrinsic brightness of the star. It is defined to be m IF the star were at a distance of 10 pc $= D$.

$$m = -2.5 \log F(D) + B$$

$$M = -2.5 \log F(10\,pc) + B$$

$$m - M = -2.5 \log\left[\frac{F(D)}{F(10\,pc)}\right]$$

But $F \propto 1/D^2$ $\qquad \frac{F(D)}{F(10\,pc)} = \left(\frac{10\,pc}{D}\right)^2 \rightarrow \log\left[\frac{F(D)}{F(10\,pc)}\right] = 2\log\left(\frac{10\,pc}{D}\right)$

$$m - M = -2.5 \times 2 \log\left(\frac{10\,pc}{D}\right) = +5\log\left(\frac{D}{10\,pc}\right)$$

$$\boxed{m = M + \underbrace{5\log\left(\frac{D}{10\,pc}\right)}_{\text{distance modulus}}}$$

As D decreases, star becomes brighter and m decreases.

M is a surrogate for the luminosity of the star.
Consider two stars at the same distance D. For them,

$$m_2 - m_1 = M_2 - M_1 = -2.5\log\left(\frac{F_2}{F_1}\right)$$

But, in this case, $\frac{F_2}{F_1} = \frac{L_2}{L_1}$: $\quad M_2 - M_1 = -2.5\log\left(\frac{L_2}{L_1}\right)$

If we are measuring both magnitude & luminosity over all wavelengths, then, letting $1 = $ sun, $\quad M_{bol} - M_{sun} = -2.5\log\left(\frac{L_{bol}}{L_\odot}\right)$

$$\boxed{M_{bol} = -2.5\log\left(\frac{L_{bol}}{L_\odot}\right) + M_{sun}} \qquad M_{sun} = +4.74$$

Filters and Bolometric Correction

In addition to bolometric M and m, we can also measure fluxes in wideband filters. Most common are

U	3650 Å	
B	4400 Å	all with width of ~100 Å
V	5500 Å	(the book uses nm = 10 Å)

If the star is a well known type (eg, main sequence), we can correct upward to get M_{bol}:

$$BC \equiv M_{bol} - M_V$$

This negative, since $M_{bol} < M_V$

Extinction and Color Excess

If there is dust between us and the star, it will appear dimmer at a given distance D. [Before the discovery of dust by R Trumpler in the 1930's, the whole scale of the Galaxy was too large.]

Dust has two effects: extinction and reddening.

Extinction: $$m_\lambda = M_\lambda + 5\log\left(\frac{D}{10\ pc}\right) + A_\lambda$$ extinction at wavelength λ

A_λ is a positive number that varies with λ in a characteristic way. It also depends on the "amount" of intervening dust. If there is a fixed proportion of dust to gas, then it depends on the "amount" of gas. Here "amount" = <u>COLUMN DENSITY</u> $= \int n\, dl$ (units: cm^{-2})

Since dust tends to redden objects, A_λ is bigger for smaller (bluer) λ.

Write the defining equation for A_λ for two different λ's and subtract—

$$\underbrace{(m_{\lambda_1} - m_{\lambda_2})}_{C_{12}} = \underbrace{(M_{\lambda_1} - M_{\lambda_2})}_{C_{12}^0} + \underbrace{(A_{\lambda_1} - A_{\lambda_2})}_{E_{12}\ (\text{positive})} \qquad \lambda_2 > \lambda_1$$

observed color index — intrinsic color index — color excess

denoted $(B-V)$, $(U-B)$, etc — $(B-V)_0$ etc

Note that both $(B-V)$ and $(B-V)_0$ give the ratio of fluxes

> A star with numerically larger $(B-V)$ etc is redder. [If m_B is larger, the B-flux is lower...]

$$\boxed{C_{12} - C_{12}^0 = E_{12} > 0}$$

If we have a spectrum for the star, we can look up C_{12}^0. From observed C_{12}, we find E_{12}. This must be proportional to the column density of dust.

Interstellar Extinction Curve

In fact, both E_{12} and A_{λ_1} (for a fixed λ_2) are proportional to the column density of the dust. Their ratio, A_{λ_1}/E_{12}, is independent of the column density, but tells us the ability of the dust to absorb light at wavelength λ_1.

1. Convention: Fix "1" = B, "2" = V and take the ratio of $\frac{A_\lambda}{E_{B-V}}$ for various values of λ.

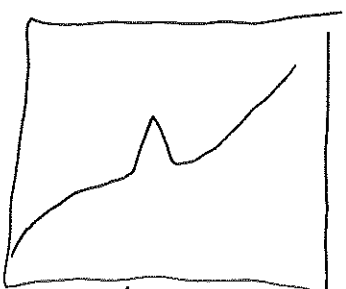
2. Convention: Alternatively, take the ratio $\frac{E_{\lambda-V}}{E_{B-V}}$ for various λ. Either ratio measures the relative extinction as a function of λ.

What is the relation between the two?

$$\frac{E_{\lambda-V}}{E_{B-V}} = \frac{A_\lambda}{E_{B-V}} - \frac{A_V}{E_{B-V}} = \frac{A_\lambda}{E_{B-V}} - R,$$

where $R \equiv \frac{A_V}{E_{B-V}} = 3.1$ for normal ISM grain

"ratio of total to selective extinction"



(my Fig 2.7) HANDOUT

In the extinction curve, for short λ ($\ll a$) $A_\lambda \propto \frac{1}{\lambda}$ (Mie theory)

The bump at 2200 Å is commonly attributed to graphite. [Dust is mainly composed of silicates plus mantle of H_2O, CH_4, SiO ..]

Extinction and Opacity

When light passes through dusty gas, the intensity is diminished:

$$I_\lambda \propto e^{-\tau_\lambda} \qquad \tau_\lambda \text{ is the optical depth}$$

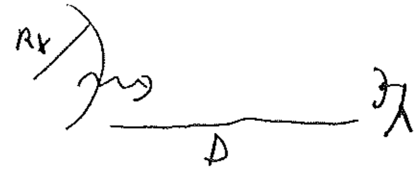
τ_λ is related to the column density of dust. It is a nondimensional quantity:

$$\tau_\lambda \equiv \int \rho k_\lambda ds \qquad \underbrace{\frac{gm}{cm^3}\ \frac{cm^2}{gm}}_{=1/\text{Labsorption}}\ cm$$

The quantity κ_λ is the opacity of the gas. (cm^2/gm)

Clearly, τ_λ must be related to A_λ:

$$F_\lambda(D) = F_\lambda(R_*)\left(\frac{R_*}{D}\right)^2 \circledast \quad e^{-\tau_\lambda}$$



or

$$-2.5\log F_\lambda(D) = -2.5\log F_\lambda(R_*) + 5\log\left(\frac{D}{R_*}\right) + 2.5(\log e)\,\tau_\lambda$$

Suppose the same star were at 10 pc, w/ no extinction

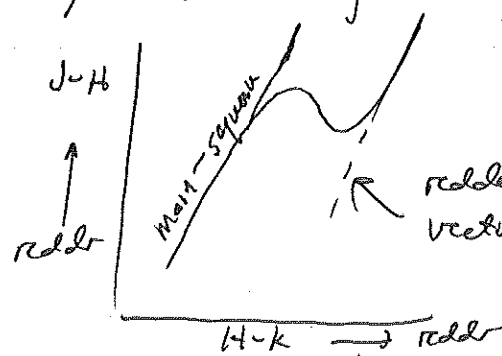
$$-2.5\log F_\lambda(10\,pc) = -2.5\log F_\lambda(R_*) + 5\log\left(\frac{10\,pc}{R_*}\right) \qquad \underline{\text{subtract}}$$

$$-2.5\log F_\lambda(D) = -2.5\log F_\lambda(10\,pc) + 5\log\left(\frac{D}{10\,pc}\right) + \underbrace{2.5(\log e)\,\tau_\lambda}_{A_\lambda}$$

$$\boxed{A_\lambda = 2.5(\log e)\,\tau_\lambda = 1.086\,\tau_\lambda}$$

Color – Color Diagrams

It is informative to plot one measure of color against another. This can tell us something about the stars themselves, & not just the amount of intervening dust.



my Fig 4.1 (Handout)

This is often done in the near-IR

- J (1.25 μm)
- H (1.60 μm)
- K (2.22 μm)

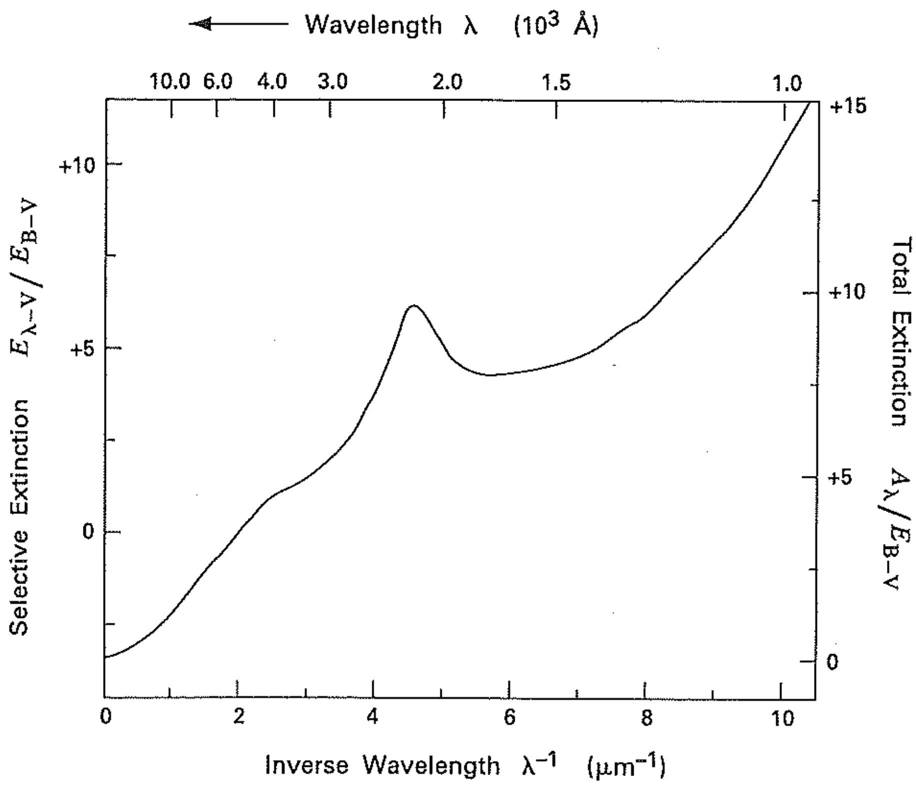
[1 μm = 10^4 Å]

Main-sequence stars that are reddened should lie within the "reddening vectors."

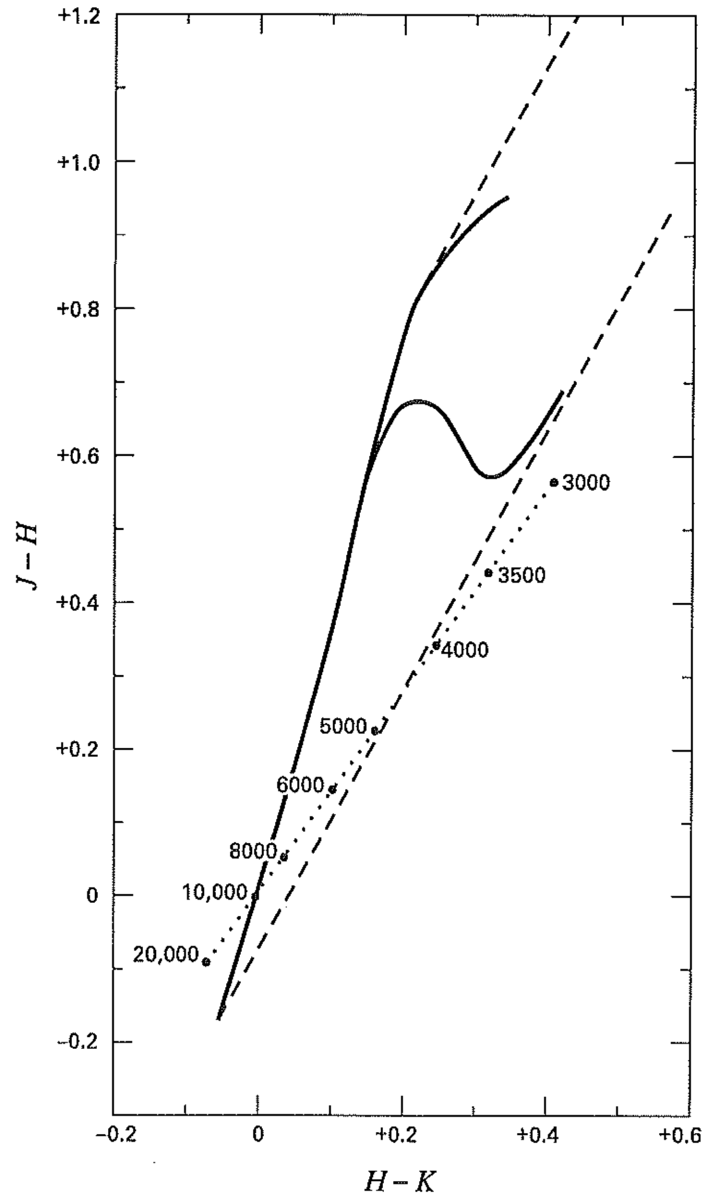
In fact, if we observe young clusters, we find many stars outside this band.

These stars have intrinsic (circumstellar) reddening, rather than interstellar reddening. Such intrinsic reddening is due to dust within the system (star, galaxy...) itself.

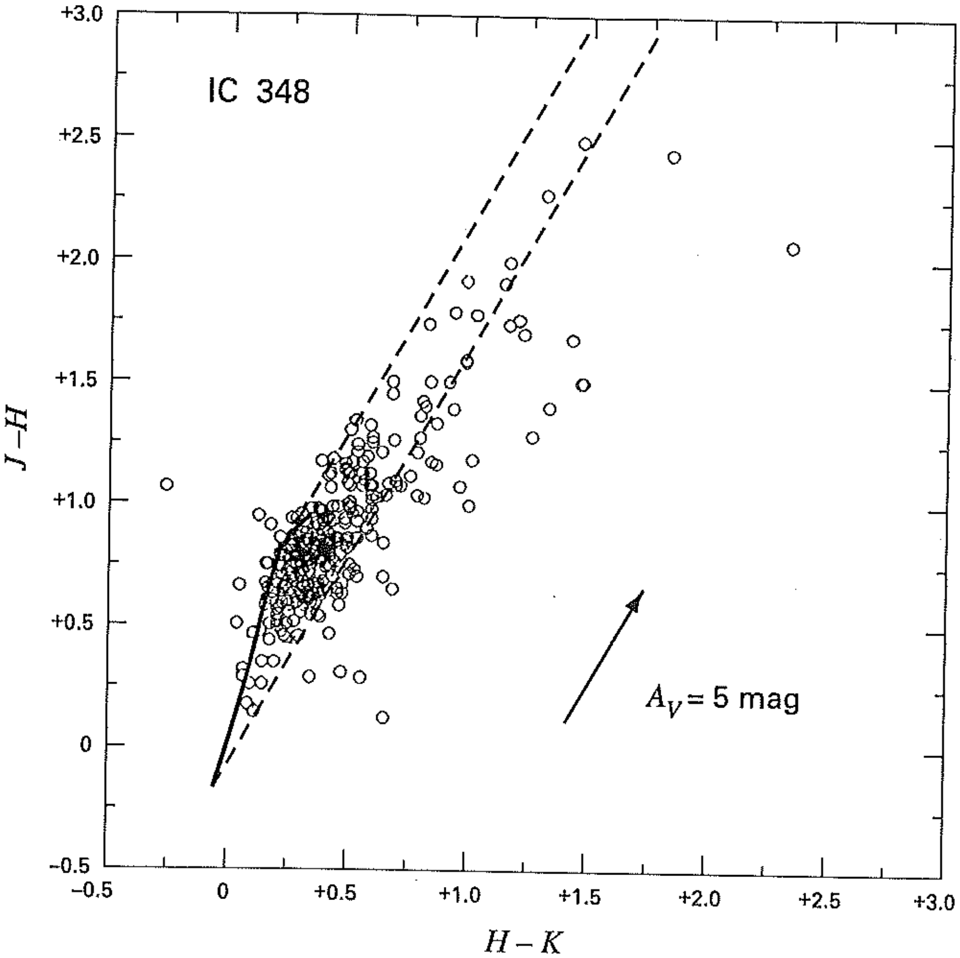
and my Fig 4.2 (Handout)



Wavelength λ (10³ Å)
10.0
6.0
4.0
3.0
2.0
1.5
1.0
+10
+5
0
Selective Extinction $E_{\lambda-V}/E_{B-V}$
+15
+10
+5
0
Total Extinction A_λ/E_{B-V}
0
2
4
6
8
10
Inverse Wavelength λ^{-1} (μm^{-1})



+1.2
+1.0
+0.8
+0.6
+0.4
+0.2
0
-0.2
J − H
−0.2
0
+0.2
+0.4
+0.6
H − K
3000
3500
4000
5000
6000
8000
10,000
20,000



IC 348
$A_V = 5$ mag
$J-H$
$H-K$
+3.0
+2.5
+2.0
+1.5
+1.0
+0.5
0
−0.5
−0.5
0
+0.5
+1.0
+1.5
+2.0
+2.5
+3.0