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MAPPING OF NEUTRAL HYDROGEN IN GALAXIES BY APERTURE SYNTHESIS TECHNIQUES

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SUMMARY

The problems of observing neutral hydrogen in external galaxies with high angular resolution are discussed. The operation of an aperture synthesis telescope with a cross correlation spectrometer giving an angular resolution better than 1′ arc with a velocity resolution of 39 km s^{-1} is described.

1. INTRODUCTION

The Cambridge Half-Mile radio telescope comprises two 30-ft (9-m) diameter paraboloids forming a variable spacing interferometer on an east–west baseline. It employs earth rotation synthesis to achieve mapping with a maximum resolution of 47″ arc at 1420 MHz. The telescope was designed with several astronomical programmes in mind, one of which was high resolution mapping of neutral hydrogen in nearby galaxies in the 21-cm spectral line; this paper describes the design and operation of the telescope for that purpose.

The principles of mapping areas of sky using earth rotation synthesis have already been described for the One-Mile radio telescope (Elsmore, Kenderdine & Ryle 1966). The particular features of the Cambridge Half-Mile radio telescope and its application for the mapping of polarized continuum radiation have also been described (Baldwin *et al.* 1970) and no further discussion of these aspects of the instrument will be given here.

Factors affecting the overall design of an instrument for hydrogen line observations are discussed in Section 2 and the particular spectrometer at present in use in Section 3. Methods of calibration and the reduction of observations are given in Sections 4 and 5.

2. DESIGN CONSIDERATIONS

The principal factors to be considered in the design of a telescope and spectrometer for the study of H I in nearby galaxies are the maximum angular resolution, limiting sensitivity, frequency resolution of the spectrometer and the total quantity of data to be collected and reduced. These will be discussed in relation to the known physical parameter of nearby late type spiral galaxies.

(a) *Angular resolution*

The optical angular diameters of these galaxies within a distance of 5 Mpc lie in the range 20–200 arc min and the corresponding angular separation of the spiral arms are from 1–10 arc min. Since the resolution of the H I spiral arms is one of the most important requirements, a beamwidth of about 1 arc min is therefore necessary. At the present time such a resolution at 21 cm can be achieved only by the use of aperture synthesis techniques and requires interferometric baselines of ~ 500 m.

For several galaxies it has been shown already (Roberts 1967) that the H I diameters are comparable with their optical diameters. It is desirable that a galaxy should lie wholly within the envelope beam of a single element of the interferometer. This both reduces the necessary observing time to cover the galaxy and avoids difficulties due to partial resolution of the galaxy at even the smallest possible spacing of the elements. To satisfy this criterion the maximum diameter of the paraboloids would be 3 m for M31 and 10–30 m for other nearby galaxies.

(b) *Sensitivity*

The theoretical limiting sensitivity in maps made using earth rotation synthesis with a two element interferometer on an east–west baseline may be derived by the following simplified argument.

For an interferometer comprising two paraboloids of diameter d and having a maximum separation D, the width, ϕ, between half-power points of the beam of the individual paraboloids is

$$\phi = \frac{1 \cdot 2\lambda}{d}$$

and the width, θ, between half-power points in right ascension of the synthesized beam is

$$\theta = \frac{\lambda}{1 \cdot 28D}.$$

The corresponding beamwidth in declination is θ cosec δ. The numerical constants are typical of those actually achieved with suitable feed systems for the paraboloids and weighting of the observations in the synthesized aperture plane.

Consider a solid angle of sky equal in size to the synthesized beam and having a brightness temperature T_b. Then the aerial temperature, T_a, due to such a source is obtained by diluting T_b by the ratio of the envelope beam area to the synthesized beam area. Taking this ratio to be $(\theta/\phi)^2 \operatorname{cosec} \delta$, then $T_a = T_b 2\eta(\theta/\phi)^2 \operatorname{cosec} \delta$ for two paraboloids each of efficiency η. For an interferometer using a correlation receiver and having a system noise T_s and bandwidth B, the r.m.s. noise in terms of aerial temperature after a total integration time t is:

$$T_{\text{r.m.s.}} = T_s \left(\frac{2}{Bt}\right)^{1/2} \quad \text{(e.g. Kraus 1966)}$$

The separation of successive interferometer spacings is usually taken to be $\approx 2d/3$, thereby enabling the observations to be analysed to the half power points of the envelope beam without introducing grating responses from sources in other parts of the beam. In this case t comprises $1 \cdot 5\ D/d$ periods of 12 hr.

The integration time used assumes equal weighting of all time intervals in the observations. In practice a gaussian weighting truncated at 30 per cent was used to give a satisfactory synthesized beam shape, resulting in a 7 per cent increase in the minimum observable brightness temperature.

Equating the r.m.s. noise to the aerial temperature, we derive the brightness temperature, T_b, of a source giving a signal equal to the r.m.s. noise

$$T_b 2\eta(\theta/\phi)^2 \operatorname{cosec} \delta = T_s \left(\frac{2}{B\ 1{\cdot}5D/d \times 12 \times 3600}\right)^{1/2} \times 1{\cdot}07,$$

i.e.

$$T_b = \frac{T_s}{168}(\phi/\theta)^{3/2}\frac{\sin\delta}{B^{1/2}}$$

taking

$$\eta = 0{\cdot}6$$

or alternatively

$$T_b = \frac{T_s}{160}.\frac{N^{3/2}}{B^{1/2}}.\sin\delta$$

where N is the number of interferometer spacings used.

A more rigorous treatment taking account of the detailed shape of the beams gives a result within 10 per cent of that obtained here. Observations of nearby late type spiral galaxies made with beamwidths of 10–20 arc min have shown maximum brightness temperatures of $\sim 30\,^\circ$K over bandwidths of ~ 100 KHz. To obtain an adequate signal to noise ratio over much of a galaxy it is clear than an r.m.s. noise equivalent to a brightness temperature T_b of about $1\,^\circ$K is desirable. For sample values of $T_s = 100\,^\circ$K and $B = 100$ KHz then $T_b = 1\,^\circ$K is achieved for $N = 65$. The actual design chosen for the telescope was influenced by the availability of 9-m paraboloids at low cost and the possibility of siting them on the existing rail track of the One-Mile radio telescope. The maximum separation of the paraboloids is 2400 ft ($N = 120$), giving an angular resolution of 47″ arc at a wavelength of 21 cm. In the initial observations with the telescope the system noise was $\sim 250\,^\circ$K and hence the sensitivity was expected to be worse than that outlined above. However, since the minimum detectable value of T_b varies as $\theta^{-3/2}$, a relatively small reduction in the angular resolution used could give an adequate limiting sensitivity.

(c) *Overall bandwidth and frequency resolution*

Rotation curves of normal spiral galaxies derived from optical work show that the largest range of velocity expected in a galaxy is approximately ± 300 km s^{-1}. At the frequency of the 21-cm hydrogen line this corresponds to Doppler shifts of $\pm 1{\cdot}5$ MHz and it is desirable that the whole of this range should be observable.

The maximum useful resolution in frequency is dependent on the angular size of the synthesized beam, since in general the H I radial velocities vary in a systematic way across the face of a galaxy. Fig. 1 illustrates two beams superimposed on the disk of a model galaxy. In the beam at A, on the major axis, the variation of circular velocity with radial distance in the galaxy is the main factor determining the range of velocities present in the synthesized beam. In the beam at B, on the minor axis, the spread in velocity is due to variation across the beam of the line of sight component of the circular velocity. For both the cases A and B there will also be dispersions in velocity due to random motions of the gas and to variation of the radial velocity of the H I at different distances along any line of sight. Estimates

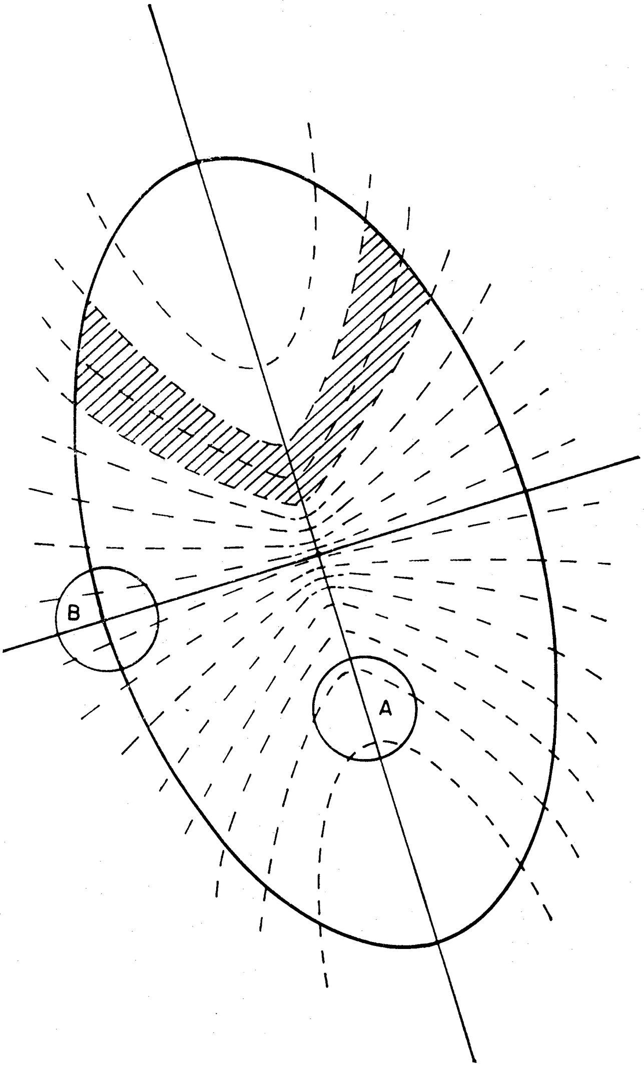

Fig. 1. *Synthesized beams superimposed on a model rotating disk galaxy. Dashed lines: lines of constant radial velocity; circles: synthesized beams.*

of the relative importance of these effects for M31 observed with a 1 arc min beam (≡200 pc at the distance of M31) are presented below.

	Near nucleus		Outer parts of M31	
	Δv km s^{-1}	$\Delta\nu$	Δv km s^{-1}	$\Delta\nu$
Variation of circular velocity with radius	0–300	0–1·5 MHz	0–2	0–10 kHz
Variation of line of sight component of circular velocity	0–300	0–1·5 MHz	0–3	0–15 kHz
Random motions	Say 10	50 kHz	Say 10	50 kHz
Line of sight	Say 300	1·5 MHz	Say < 10	< 50 kHz

From the values in the table it is seen that, in the outer parts of M31, the line broadening due to averaging over the beam is probably smaller than that due to random motions in the H I. A frequency resolution better than 50 kHz might therefore be useful. For more distant galaxies the spread of velocities inside the synthesized beam becomes progressively more important.

(d) *Computational problems*

For an instrument having angular and frequency resolutions as outlined above, the time required for the reduction of the observations on even a fast digital computer is a serious design consideration. Frequency resolution may be obtained by use of a bank of filters, so that each of say M frequency bands can be observed simultaneously. There are, however, advantages in using a correlation spectrometer in which, at each sampling time, the spectrum can be obtained by a Fourier transform. The principal computations involved are then:

(*i*) *Frequency Fourier Transform.* If the spectrometer is of the cross-correlation type having an output for each of $2M$ different delays, then a Fourier Transform of these outputs to give a spectrum comprising values at M different frequencies is required for each data sample.

(*ii*) *Spatial Fourier Transform.* For each of the M frequencies a two-dimensional Fourier Transform is required of the sampled aperture plane. This data is obtained from 12-hr observations at each of N spacings of the movable aerial.

To give adequate oversampling of the primary data, $\sim 6N$ samples are required in 12 hr at spacing number N. It is convenient if the data for all the spacings are sampled at the same rate. Although this leads to a large degree of redundancy at the shorter spacings, the total amount of data so recorded is only twice the minimum required.

The frequency Fourier Transform of the $2M$ outputs at each sampling time requires of order $2M \log_2 2M$ operations using fast Fourier techniques and $(2M)^2$ otherwise. (Here an operation consists of a complex multiplication followed by a complex addition, taking $> 50\,\mu$ secs on all but the fastest computers.) The frequency Fourier Transform must be performed on each sample of the survey and the total number of operations required is therefore $12N^2M \log_2 2M$.

The spatial Fourier Transform for each of the M frequencies may be assessed by the following simplified argument. The oversampled aperture may be regarded as an approximately $4N \times 4N$ rectangular array of samples. A two-dimensional Fourier Transform then comprises $2 \times 4N$ one-dimensional Fourier Transforms each of $4N$ points requiring a total of $8N.4N \log_2 4N$ operations. The total number of operations for all of the frequencies is therefore $32MN^2 \log_2 4N$.

Taking, as an example, observations at 100 spacings ($N = 100$) with a 100 output cross-correlation receiver ($2M = 100$) then the times calculated above are 2100 s for the frequency Fourier Transform and 7200 s for the spatial Fourier Transform. In practice it may be difficult to achieve the full benefits of fast Fourier techniques in the spatial synthesis since the actual sampling of the aperture plane is very far from being a regular rectangular array of points. Thus the frequency Fourier Transform will occupy only a small fraction of the total computing time which will be several times the minimum value of $2\frac{1}{2}$ hr calculated here. In a set of observations of this type the total number of data values to be stored is $\sim 6N^2 \times 2M$, i.e. 6×10^6 and the corresponding number of values on the output maps is similar. Such quantities of data can be stored satisfactorily on a single magnetic tape, but extensions of surveys to larger values of M and N by say a factor 10 evidently entail serious problems of data storage.

3. THE SPECTROMETER

A receiver of the cross-correlation type was chosen for the present system for reasons of low cost, simplicity of construction and maintenance, and because the computing time associated with the frequency Fourier Transform is only a small part of the total. It differs in many important respects from the digital auto-correlation spectrometers introduced by Weinreb (1963) and more closely resembles

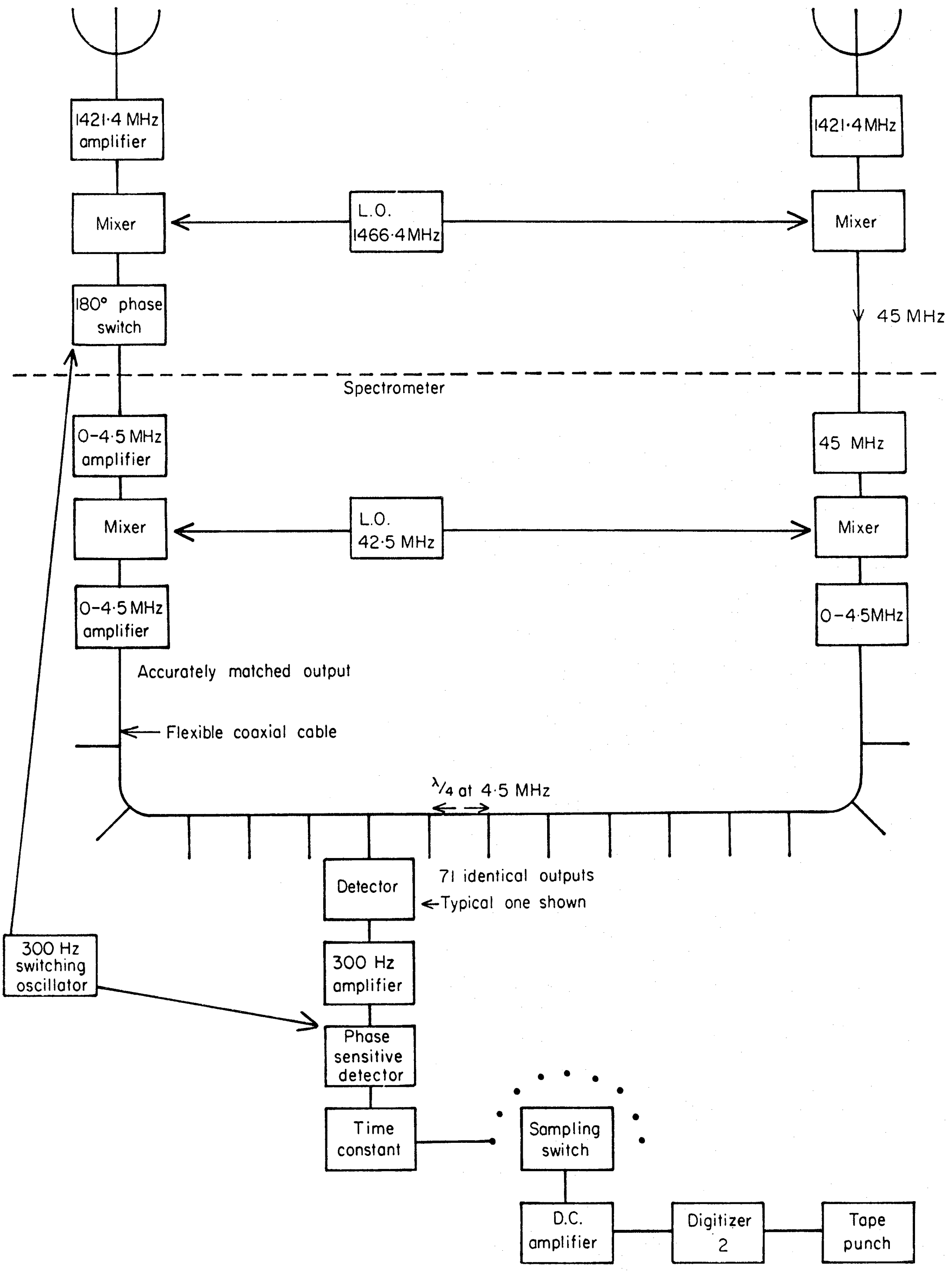

FIG. 2. *Block diagram of the spectrometer.*

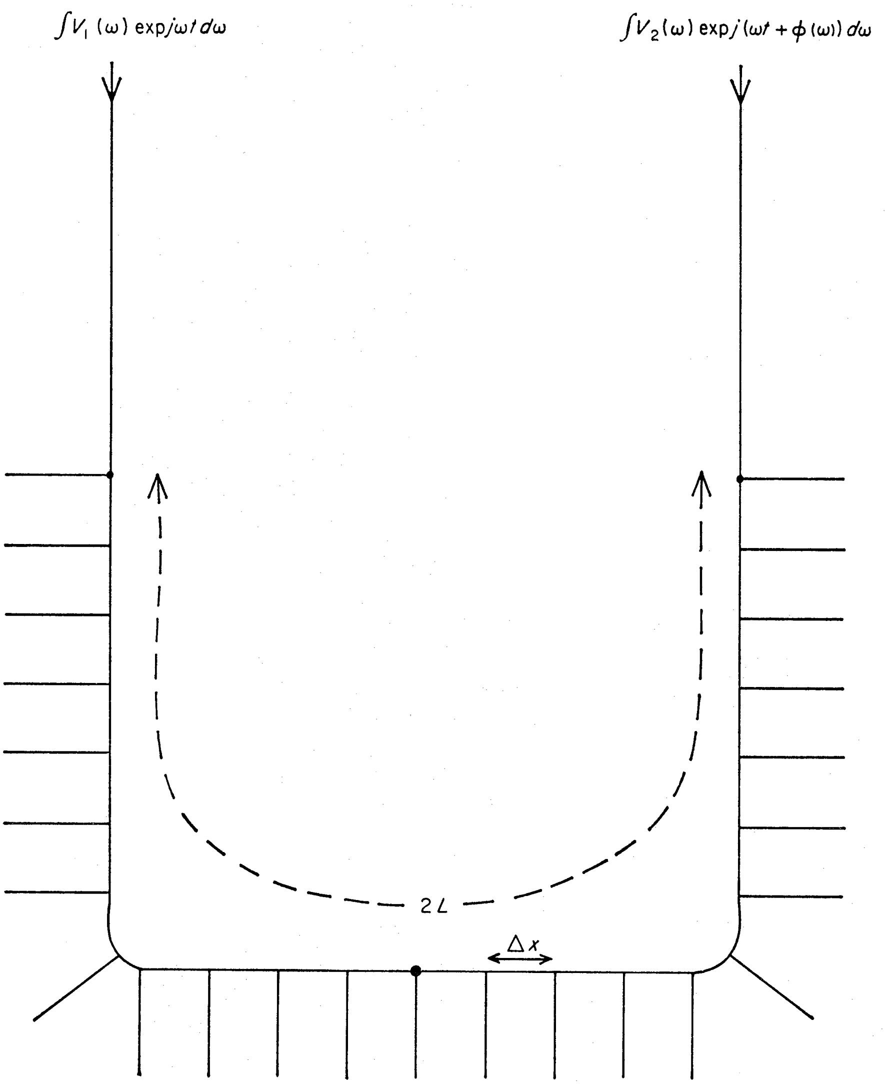

FIG. 3. *Physical quantities used in analysing the operation of the spectrometer* (*see text*).

that described by Argyle (1965). A block diagram of the spectrometer is shown in Fig. 2.

The principle on which the spectrometer operates depends on the result that, for two random time varying signals, $f_1(t)$ and $f_2(t)$, the cross-correlation function

$$\sigma(\tau) = \int f_1(t)f_2(t-\tau)\, dt$$

is the Fourier Transform of the cross spectrum of the two signals. In our case the two signals are those arriving from the two separated aerials.

The detectors, situated at equal intervals, Δx, along the delay cable, have outputs whose values are proportional to the cross-correlation function of the input signals. This may be seen by consideration of a particular detector situated at distance $m\Delta x$ from the centre of the cable of total length $2L$ (see Fig. 3). If the two inputs to the ends of the cable are

$$\int V_1(\omega)e^{j\omega t}\, d\omega$$

and

$$\int V_2(\omega) e^{j(\omega t + \phi(\omega))}\, d\omega$$

and α is the voltage attenuation constant of the cable, the operation of the 180° phase switch gives an output from the detector at the 300 Hz switching frequency:

$$\propto \mathscr{R}\left\{\left(\int V_1(\omega) \exp\left[j(\omega t - 2\pi L/\lambda - 2\pi m\Delta x/\lambda)\right] \exp\left[-\alpha(L - m\Delta x)\right] d\omega\right)^*\right.$$

$$\left.\times \left(\int V_2(\omega) \exp\left[j(\omega t - 2\pi L/\lambda + 2\pi m\Delta x/\lambda + \phi(\omega))\right] \exp\left[-\alpha(L + m\Delta x)\right] d\omega\right)\right\}$$

$$= \int V_1(\omega) V_2(\omega)[\cos \phi(\omega) \cos (2m\omega\Delta x/c) + \sin \phi(\omega) \sin (2m\omega\Delta x/c)] \times \exp(-2\alpha L)\, d\omega.$$

The two terms in the bracket correspond to the in-phase (cosine) and quadrature (sine) outputs of a conventional phase-switched interferometer multiplied by a sinusoidal variation in frequency. It may be noted that the attenuation of the outputs due to the cable delays are equal for all m. Thus the set of $(2M-1)$ outputs from the detectors at any moment are proportional to the cross-correlation function of the input signals sampled at equal intervals of time delay of $2\Delta x/c$. The Fourier Transform of such a set gives the power spectrum of the correlated signal over a bandwidth of $c/4\Delta x$ with a resolution in frequency of $c/4(M-1)\Delta x$ if equal weighting of the cable outputs is used. In practice a truncated gaussian weighting of the outputs is used to give reduced frequency sidelobes.

The parameters of the receiver at present in use are as follows:

Overall bandwidth	0–4·5 MHz
Number of detectors on delay cable $(2M-1)$	71
Half-width frequency response (equal weighting)	130 kHz
Half-width frequency response (gaussian weighting)	185 kHz

A number of factors affect the overall noise performance of the spectrometer making the final noise level greater than the theoretical value derived in Section 2(b) by a factor of 2·0. These factors are

(a) The spectrometer employs simple phase switching ($\times\sqrt{2}$);
(b) The outputs are sampled at intervals of twice the time constant ($\times$ 1·15);
(c) Poor image rejection in the second mixer ($\times$ 1·15);
(d) Attenuation in the delay cable ($\times$ 1·06).

4. CALIBRATIONS

Calibration of the instrument for hydrogen line observations was achieved by regular measurements of:

(a) The relative sensitivities of the 71 output channels;
(b) The sensitivity and collimation error of the telescope as a function of frequency.

Measurements of (a) were made using a narrow-band signal from a signal generator. The stability of the gains of the channels is such that these need to be measured only at infrequent intervals. Measurements of (b) are obtained by observations of a suitable continuum radio source of small angular size whose position and flux density is well determined. The output can be analysed to give, for each independent frequency in the band, a plot against hour angle of the amplitude and phase of the signals from the source in an exactly similar manner to that described for continuum observations by Elsmore *et al.* (1966).

5. REDUCTION OF THE OBSERVATIONS

Many stages of the reduction of the observations to give maps of an area of sky in a particular range of frequencies are similar to those previously described for continuum observations (Elsmore *et al.* 1966) and only those processes peculiar to the reduction of observations with the spectrometer will be described here. The stages of particular importance are:

(a) *Frequency Fourier Transform*

The output of the correlation receiver, the sampled cross-correlation function, is first corrected for the relative sensitivities of the output channels. It is then weighted by a gaussian function truncated at 30 per cent to give a satisfactory instrumental profile in frequency and Fourier Transformed to yield cosine and sine outputs for 36 synthesized frequencies at each sampling time.

Since observations may commonly extend over a period of several months, correction for the Earth's velocity round the Sun must be applied at this stage before combining the observations. With the frequency resolution used (39 km s^{-1}) it is sufficient to make a single correction for a 12-hr period, ignoring variations in radial velocity within that period.

(b) *Spatial Fourier Transform*

The pairs of cosine and sine terms at any one frequency for all samples in time and for all spacings of the telescopes are then processed in an exactly similar way to those derived from broad band continuum observations (Elsmore *et al.* 1966). Corrections for the instrumental sensitivity and phase at each frequency are applied at this stage, derived from measurements of the calibration source on the assumption that its continuum radiation is constant at all frequencies in the band. The effects of absorption by local galactic hydrogen can be eliminated by observations of several calibration sources. The results at the end of this stage are a set of maps of the H I radiation in different frequency bands.

(c) *Combination of results*

All of the information derived from the observations is contained in the maps obtained at 36 different frequencies but for some purposes it is not displayed in the most convenient form. Computer programs have therefore been developed to provide the following:

(1) A map of the continuum radiation including discrete sources. This is derived by averaging the cosine and sine pairs at each sample time for all frequencies believed to be free from H I radiation. The spatial Fourier Transform is then carried out on these averaged values.

(2) A map of H I radiation integrated over all frequencies present in a galaxy.

This is achieved in two different ways:

(i) The pairs of cosine and sine values for a range of frequencies containing the H I radiation can be averaged before performing the spatial Fourier Transform. This may be most useful if a wide spread of velocities of H I exist at all points in a galaxy.

(ii) Portions of the maps of H I at particular frequencies can be summed. The advantage of this process is that over any beam area in a galaxy the range of velocities of H I may be much smaller than in the galaxy as a whole. A better signal to noise ratio in the integrated H I map may therefore be achieved by summing over only those velocities believed to be present in any part of the galaxy. In choosing this range of velocities care must be exercised in respect of the possible presence of high velocity hydrogen.

The contribution of the continuum radiation as derived in (1) above may then be subtracted from the resulting maps to give a map of the integrated H I radiation.

(3) Frequency spectra of the radiation at points on a rectangular grid on the area of sky studied.

(4) A map of the velocity field in the area, the contours being iso-velocity lines. This gives a useful display of the motion of the H I only in those cases where the spectra at any point consist of a simple profile which may be represented adequately by a single value of velocity.

(5) A map of the velocity dispersion at any point in the galaxy.

The application of the telescope and the reduction techniques set out above to observations of nearby galaxies will be described in subsequent papers.

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